

Klimamodelle

Experten- und Laienmodelle

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Klimamodelle

- Klima schwankt auf allen Raum- und Zeitskalen:
 - Milankovitch / Croll Zyklen
 - Eiszeiten
 - "Kleine" Eiszeit
 - El Nino/ Southern Oscillation
 - Jahreszeiten
 - Wetterstatistik
- In diesem Vortrag werden nur Zeitskalen von höchstens wenigen Jahrzehnten behandelt (also keine Eiszeiten)
- Wir betrachten nur Klimavariationen um den gegenwärtigen Zustand herum
(also keine Oeschger/ Dansgaard+Heinrich events)

Abbildung 2.6: Raum- und Zeitskalen von Phänomenen der Ozeanzirkulation

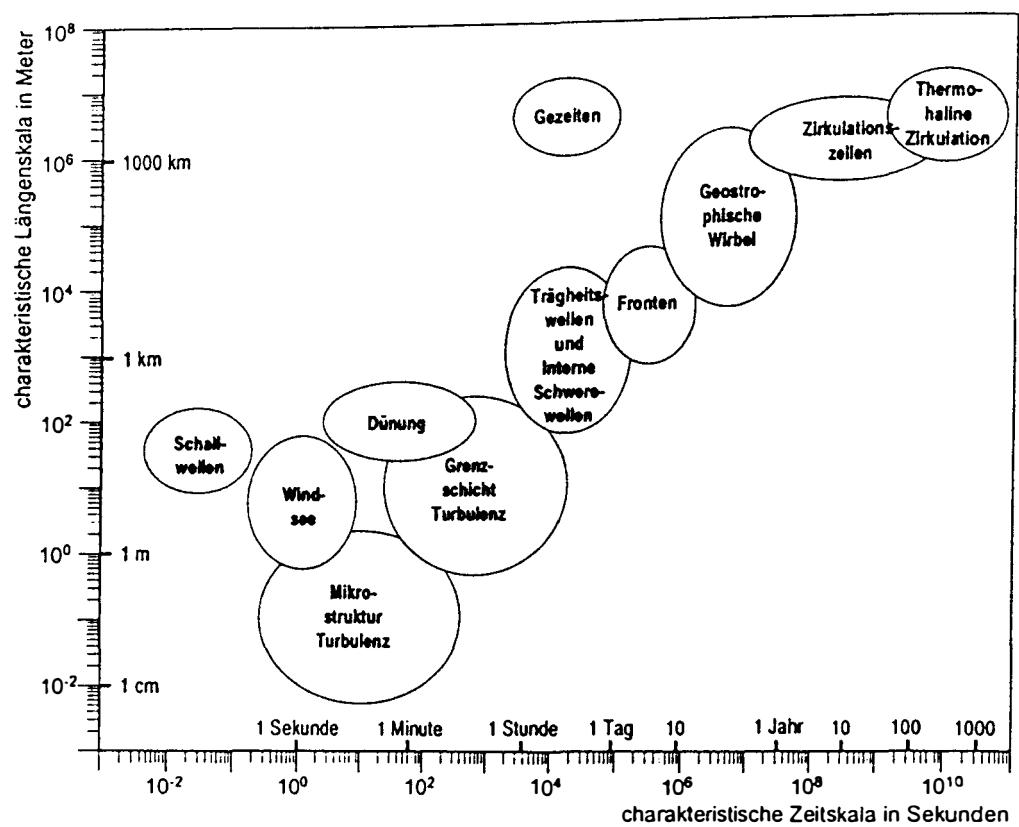
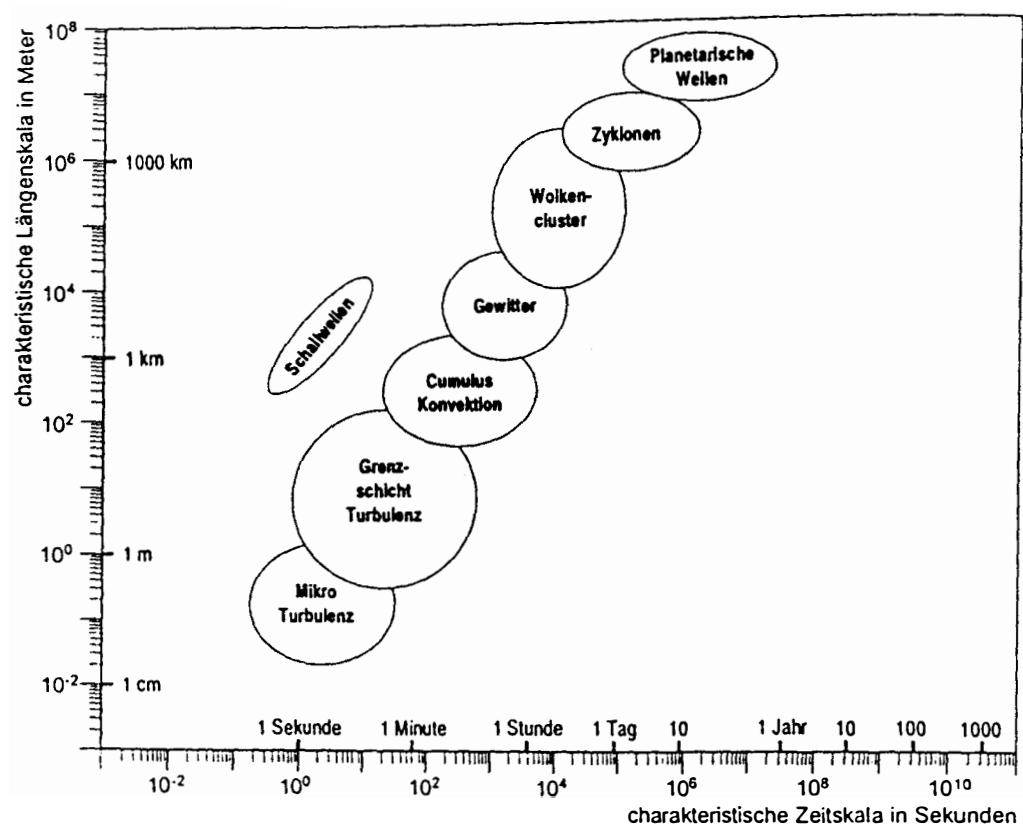


Abbildung 2.4: Größenordnungen und Zeitskalen von atmosphärischen Bewegungsvorgängen



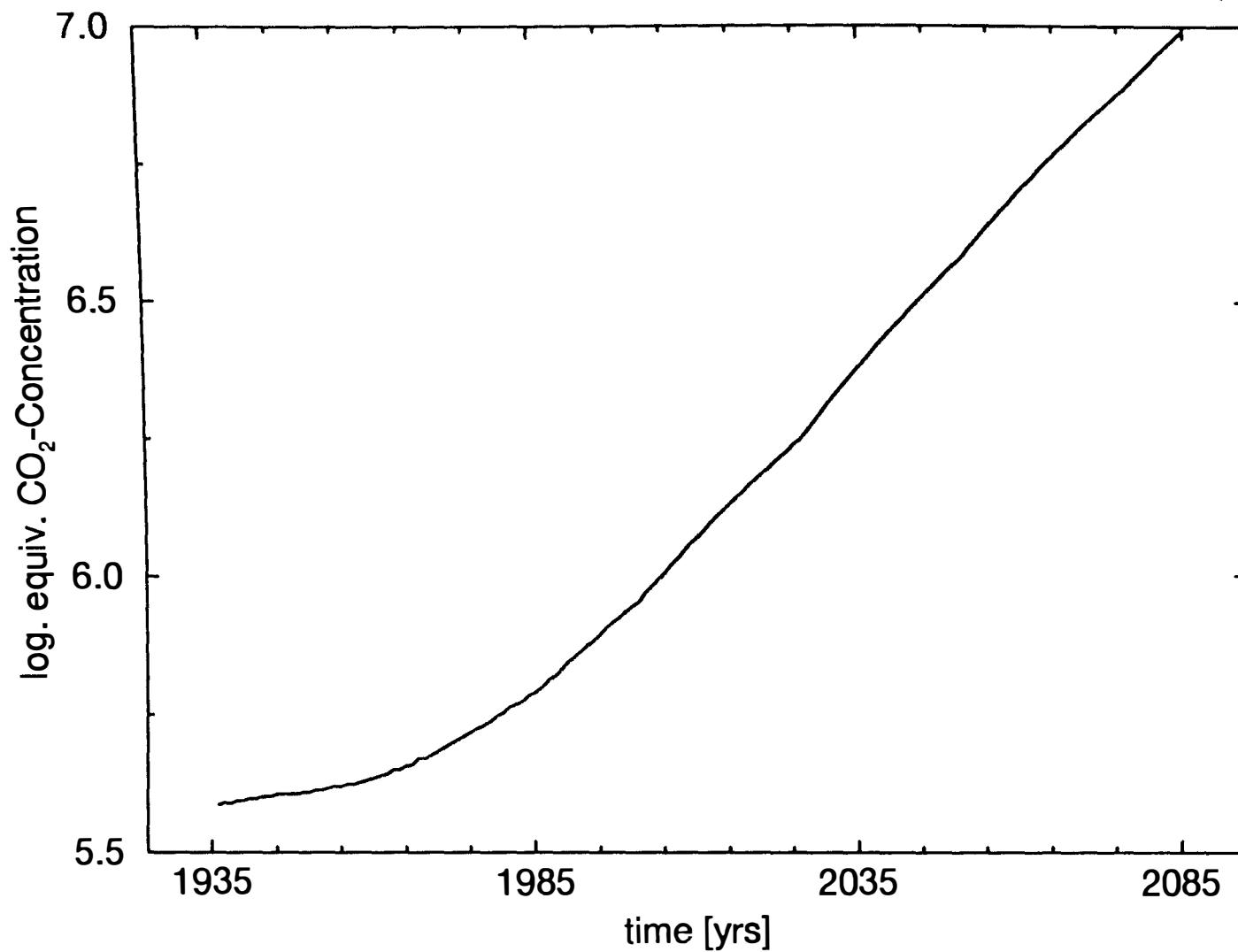
Übersicht

- Klimamodelle in der Öffentlichkeit
 - Szenarien von Klimawandel
- Klimamodelle als wissenschaftliches Instrument
 - Zirkulationsmodelle
 - Ersatzrealität; daher möglichst komplex
(wie die Realität)
 - erlauben kein unmittelbares Verständnis
 - erlauben "numerische Experimente"
 - erlauben die Untersuchung von Detailvorgängen
(keine Datenprobleme!)
- Konzeptionelle Modelle
 - Energiebilanzmodell
 - Statistische Modelle von Zirkulationsmodellen
 - erlauben "Verständnis"

Übersicht

- "Verbreiterung" von Modellen
 - Hinzufügung weiterer naturwissenschaftlicher Module (Stoffkreislauf, Vegetation)
 - Hinzufügung ökonomischer Module: Global Environment and Society Model
- Barriere: Gesellschaftsmodelle?
 - Perceived Environment and Society Model
 - Mentale Klimamodelle in der Öffentlichkeit:
Die Kempton- Studie.
Weiße Weihnachten in Zürich.

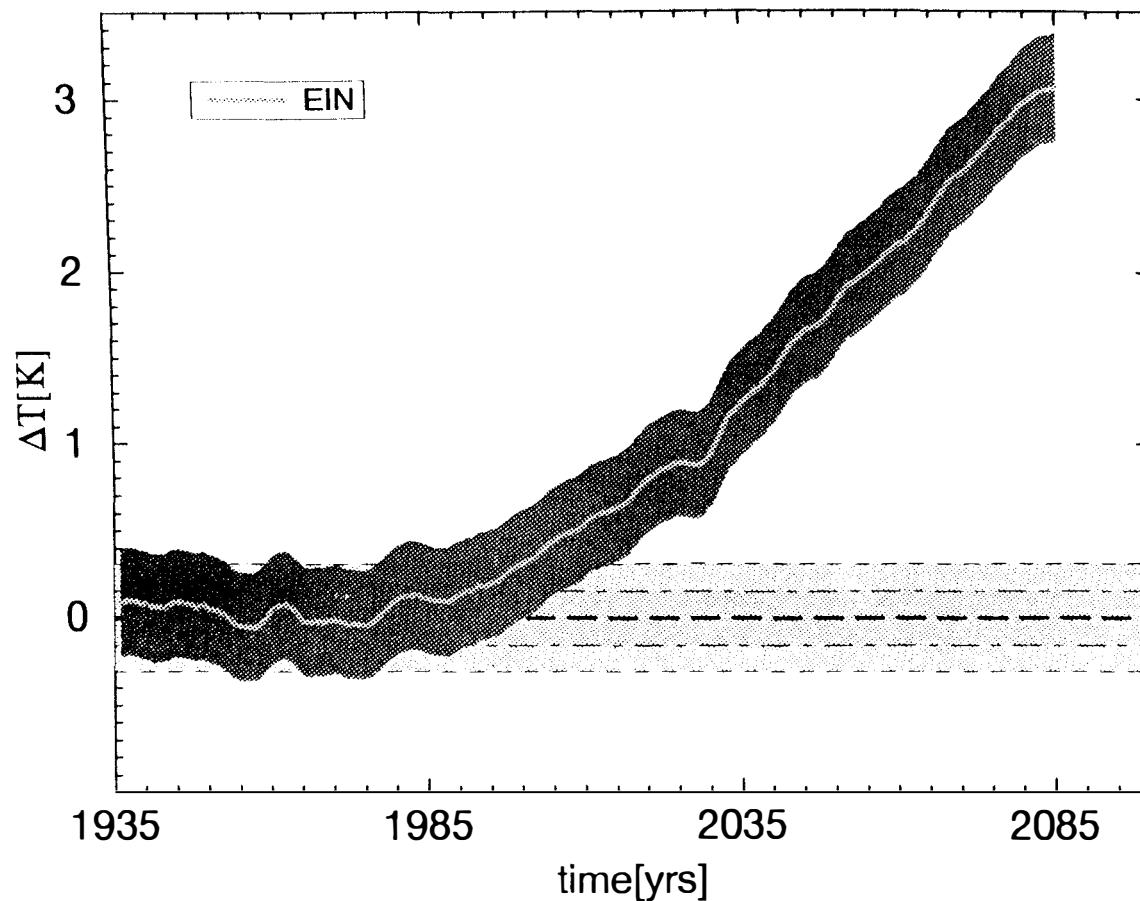
Prescribed CO_2 concentration in "ErN"-experiment



Cubasch et al, 1994

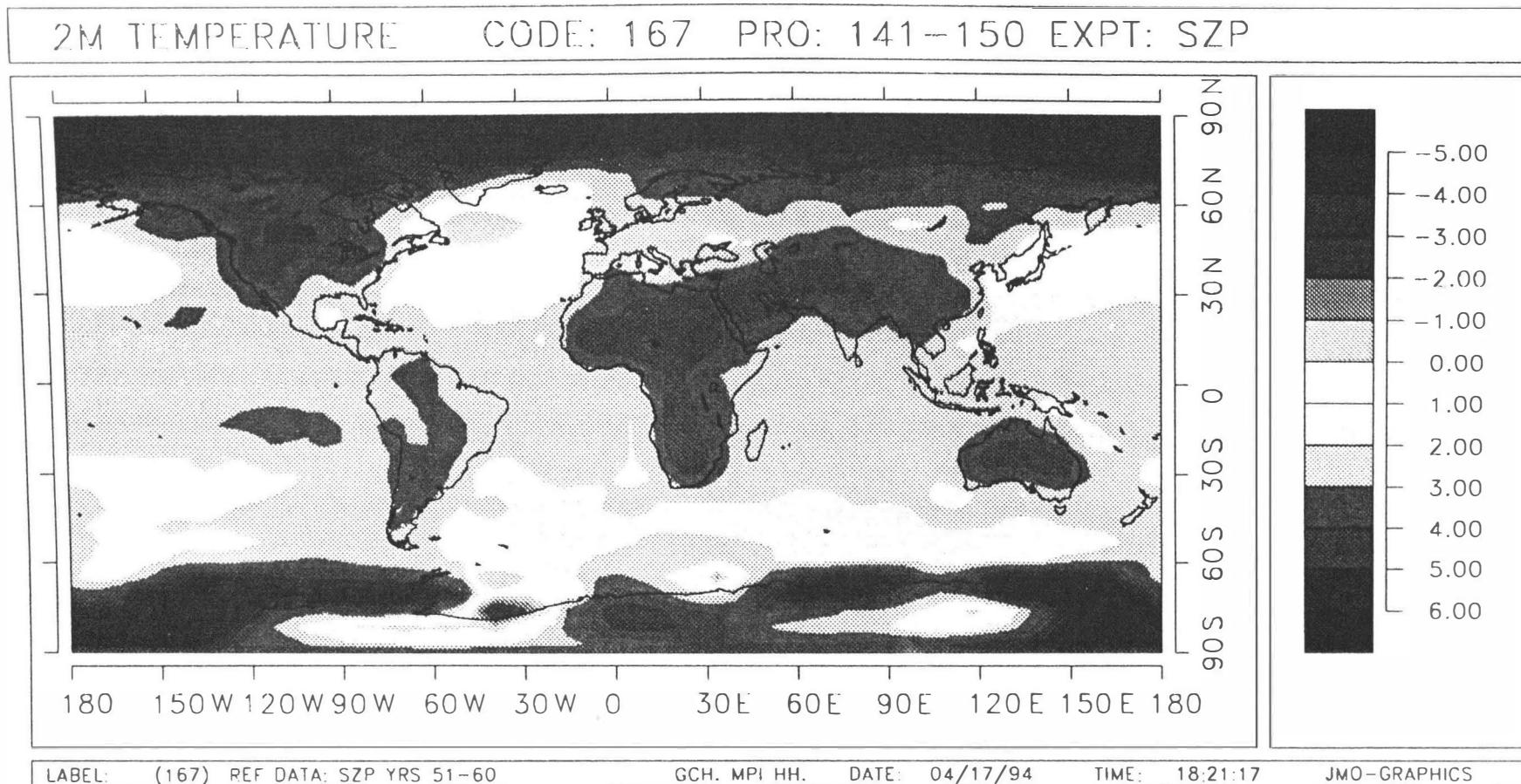
LSG/ECHAM 2m Temperature

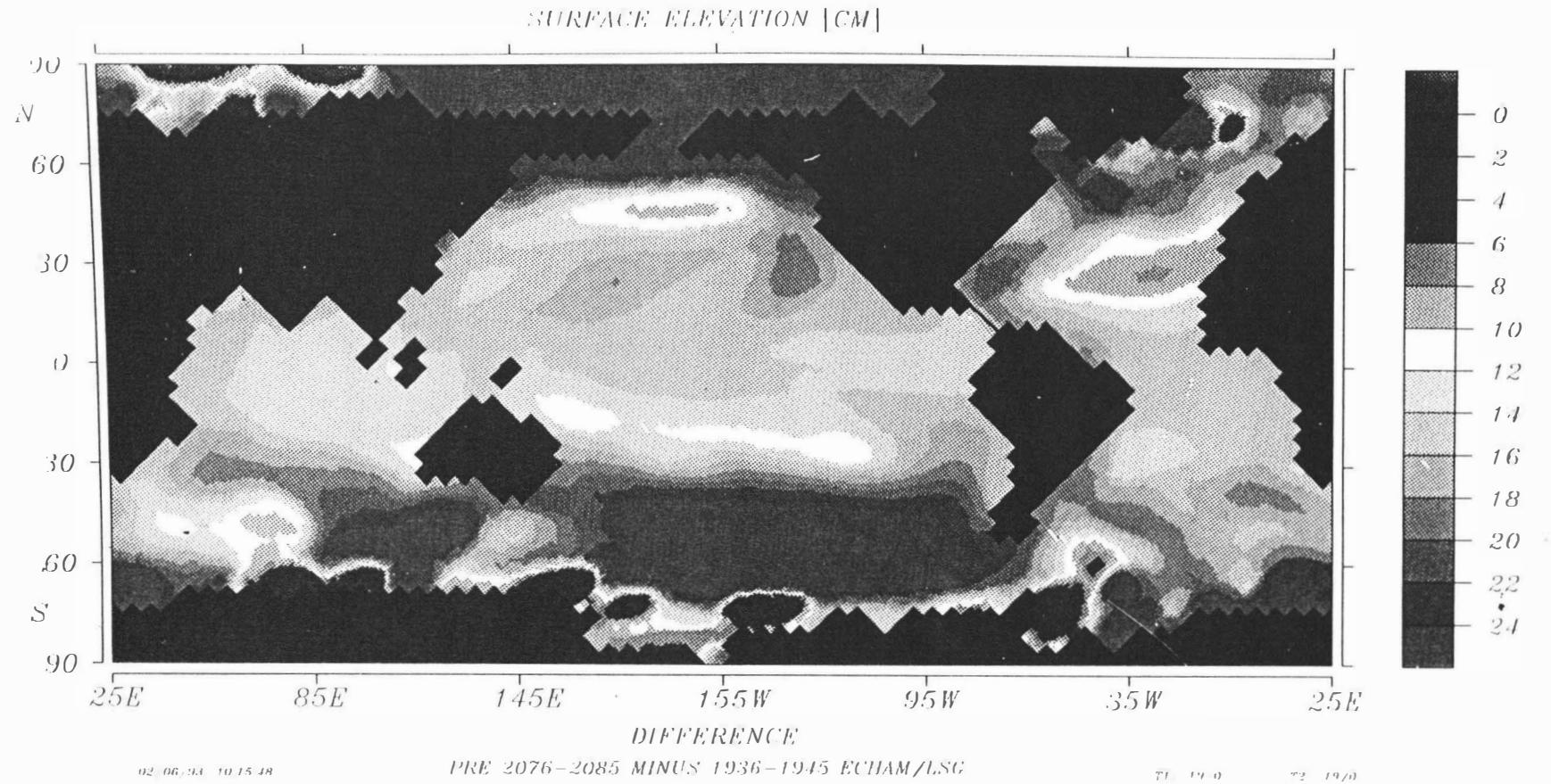
global



Cubasch et al, 1994

(2076-85) - (1986-95)



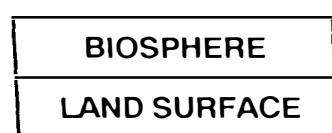


*"2076-2085" minus "1936-45"
thermal expansion
 CO_2 -forcing (Scenario A) only*

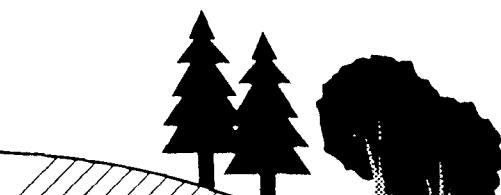
Realitätsnahe Klimamodelle ...

- ... beschreiben mit komplexen nichtlinearen Differentialgleichungen die Hydro- und Thermodynamik der Atmosphäre, und des Ozeans und stellen die Wirkungen von anderen klimarelevanten Komponenten, wie Meereis, Vegetation etc. in parameterisierter Form dar.
- ... haben sehr viele Freiheitsgrade ($> 10^6$), und erzeugen aufgrund der internen Prozesse Variabilität auf allen Raum- und Zeitskalen.
- ... sind numerische Approximationen und beschreiben daher nur die Variabilität auf aufgelösten Skalen. Vorgänge auf nichtaufgelösten Skalen werden in parameterisierter Form dargestellt. Dazu gehören Prozesse wie Strahlungsabsorption, Wolken etc.

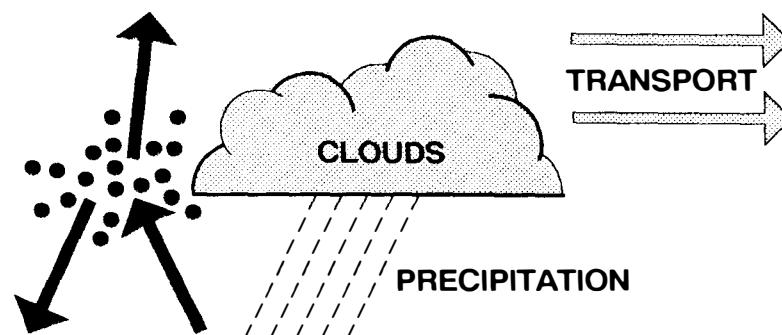
CLIMATE SUB-SYSTEMS



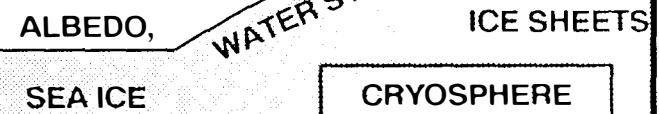
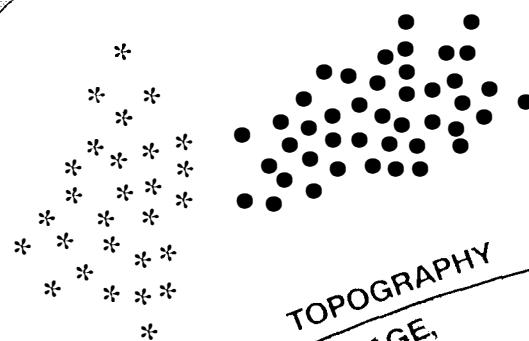
EVAPOTRANSPIRATION
ALBEDO, DRAG, CO₂



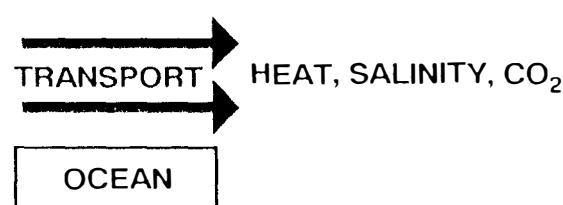
RADIATION



GREENHOUSE GASES



ALBEDO,
SEA ICE
WATER STORAGE,
ICE SHEETS



Gleichungen

- Massenerhaltung
 - trockene Luft
 - Wasser
 - Salz
 - (- Spurenstoffe)
- Energieerhaltung
(Strahlungsenergie, Wärme, mechanische Energie)
- Impulserhaltung
(aber austausch von Drehimpuls mit fester Erde)
- Zustandsgleichung
Dichte = $f(\text{Druck}, \text{Temperatur}, \text{Wassergehalt})$ Luft
= $f(\text{Druck}, \text{Temperatur}, \text{Salzgehalt})$ Ozean.

Differentialgleichungen

Diskretisierung

- Zeit: Differenzen

- Raum: Differenzen (Ozean)

Galerkin (Kugelfunktionen, Atmosphäre)

Berechnung der "komplizierten" Terme

mit Transformationsmethode auf Gauß-Gitter.

typische Auflösung: st. 20-40 min Zeitschritt

horizontal: $\Delta x = 40$

vertical: variabel

Oben: Diskrete Analysen der 850mb Temperatur (Farbskala) Unten: Diskrete Analysen 500mb Geopotentielle Höhe (Farbskala) and Luftdruck auf Meerehöhe (schwarze Isolinien)

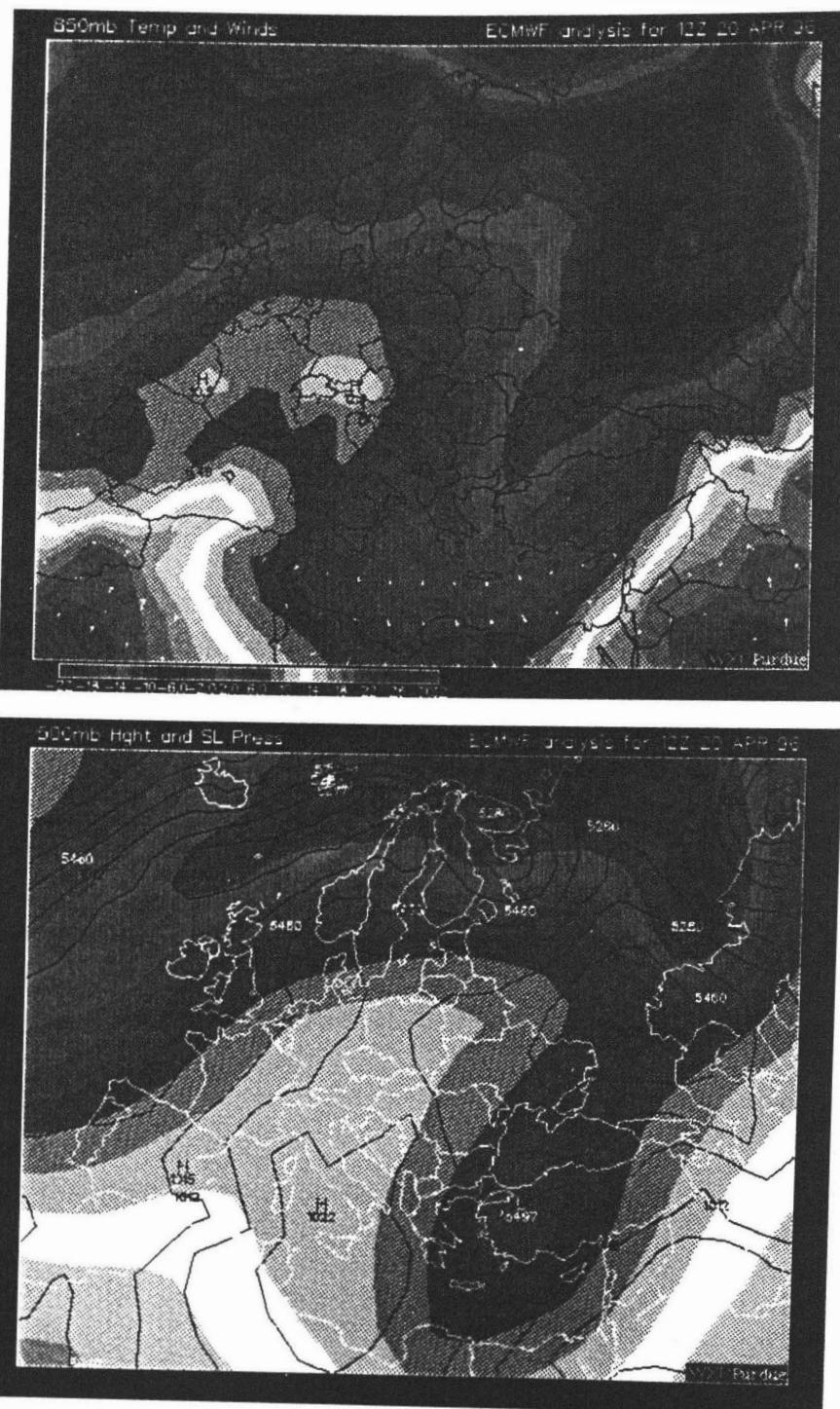
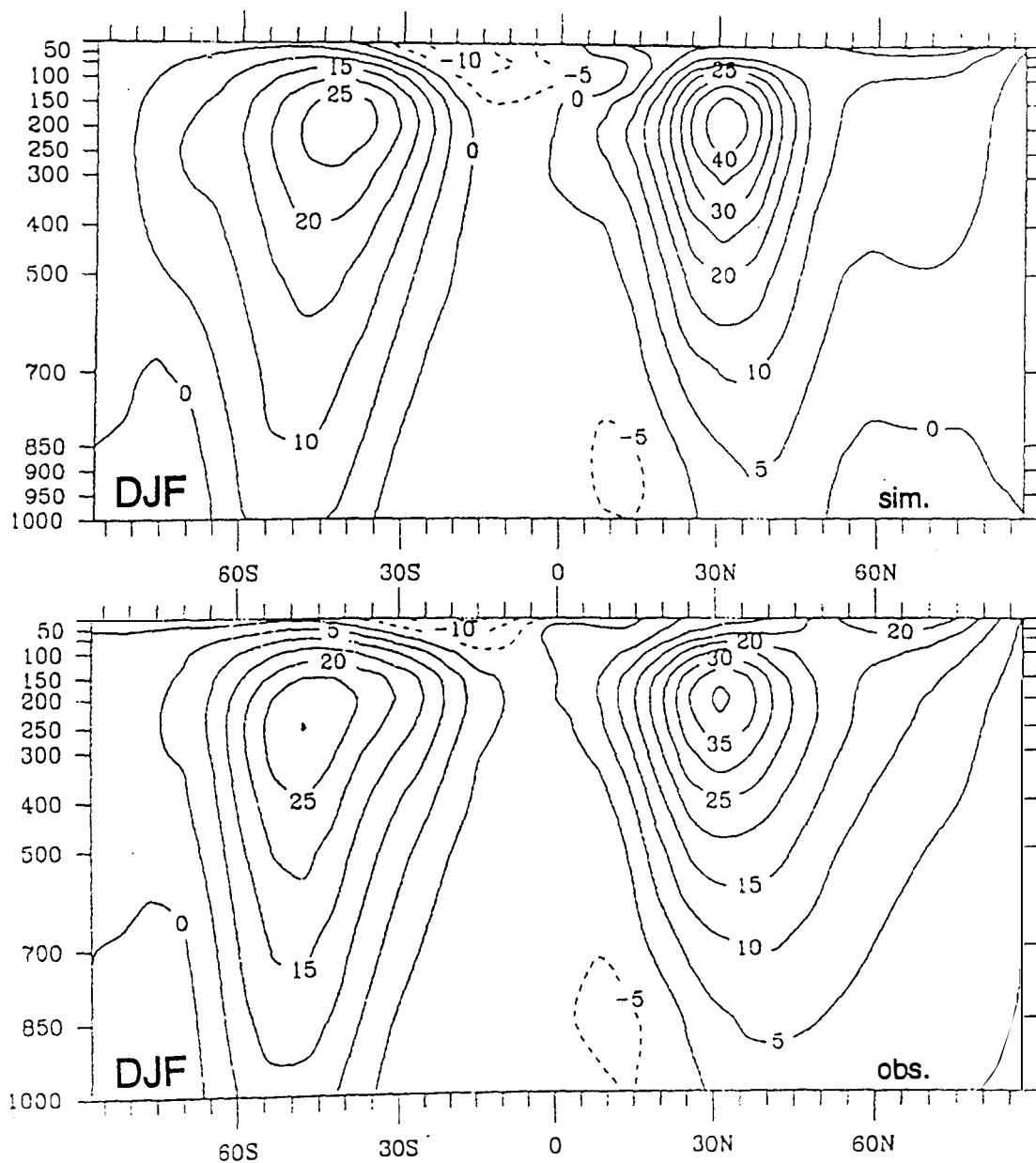


Abbildung 8.3: Oben: Vorhersage 4 Tage (20.4.96 auf 24.4.96) von 850mb Temperatur (Farbskala) und Wind (Vektorpfeile) Unten: Der tatsächlich eingetroffene Zustand (als Analysen).

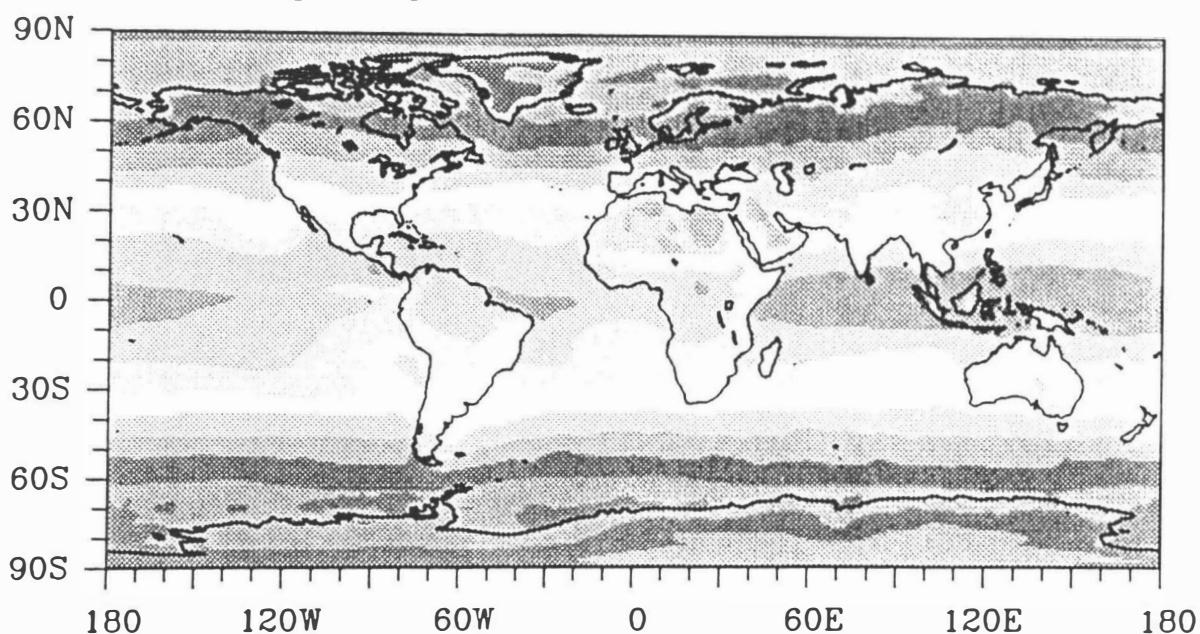




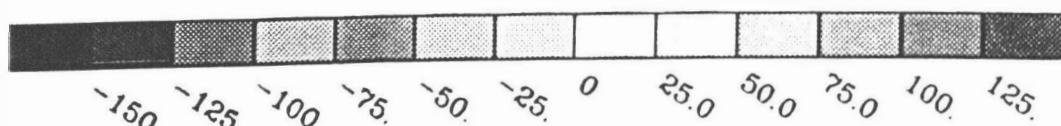
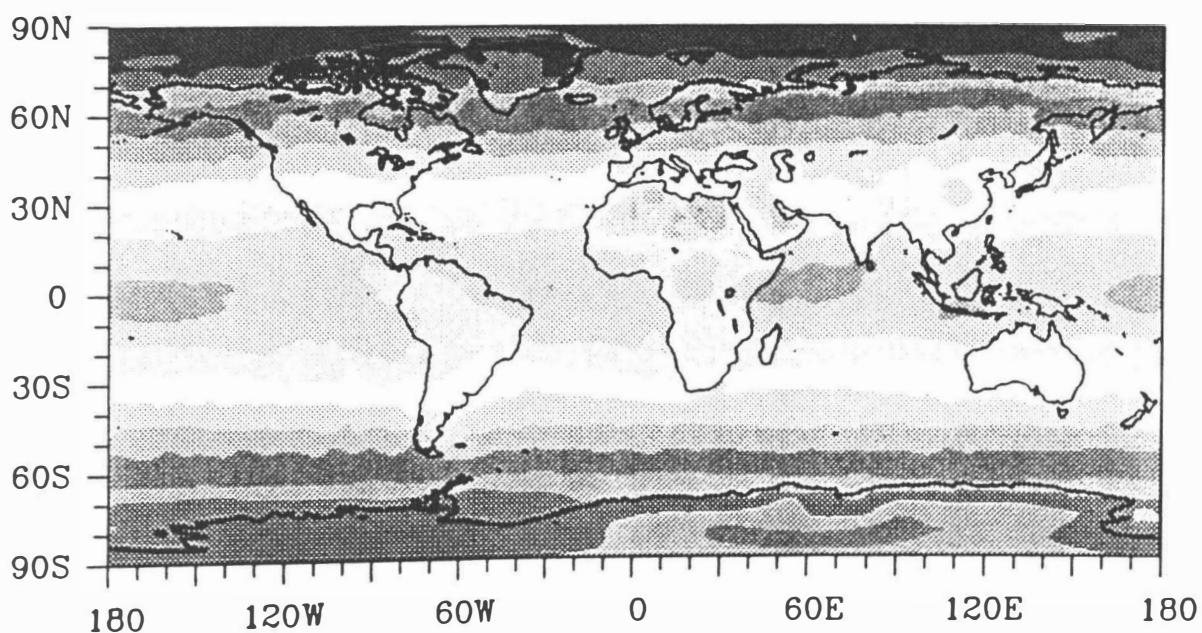
044HVSb.drw

Net [W/m²] annual mean 1985–1989

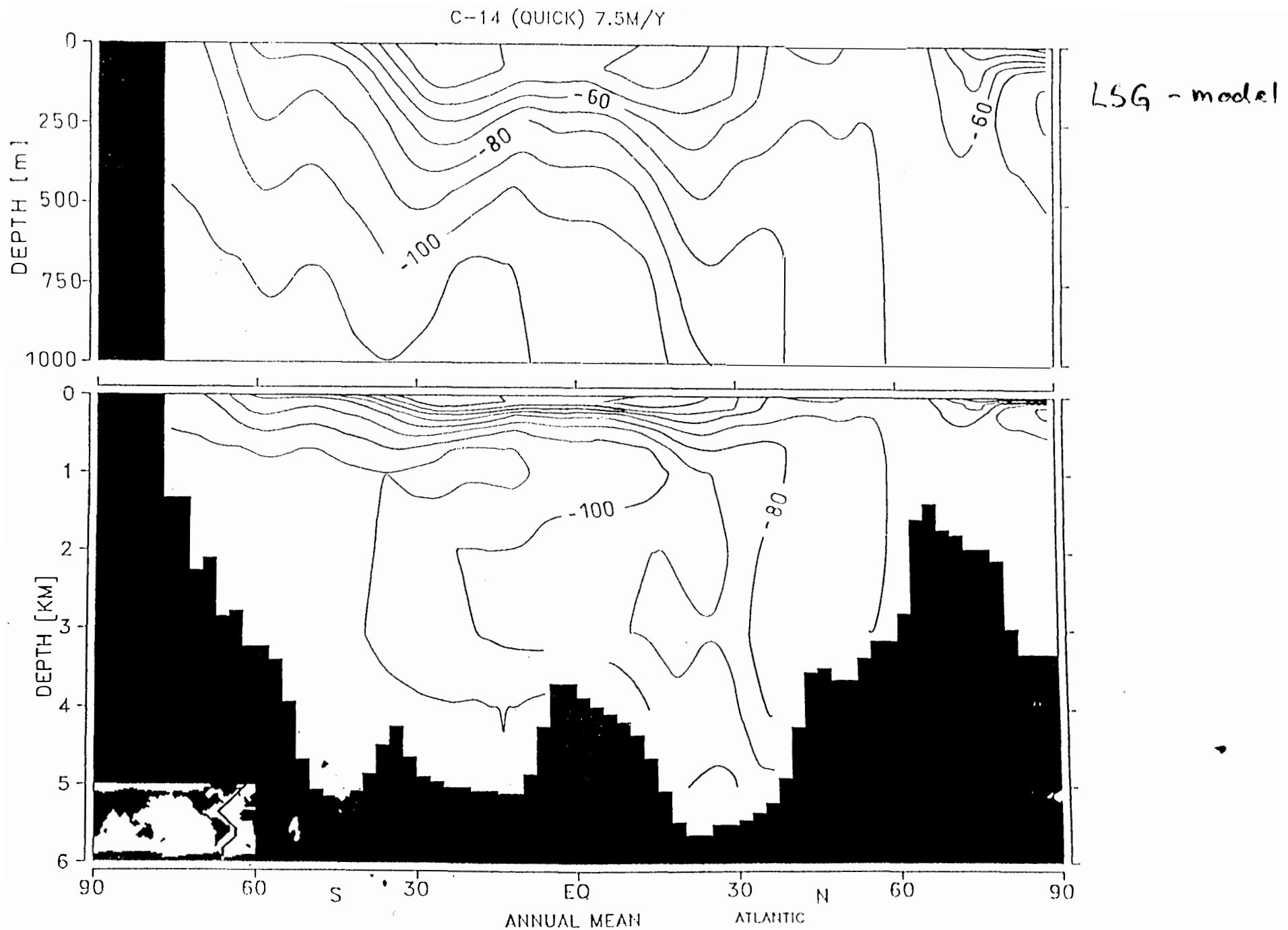
ERBE



ECHAM



Chen, 1994

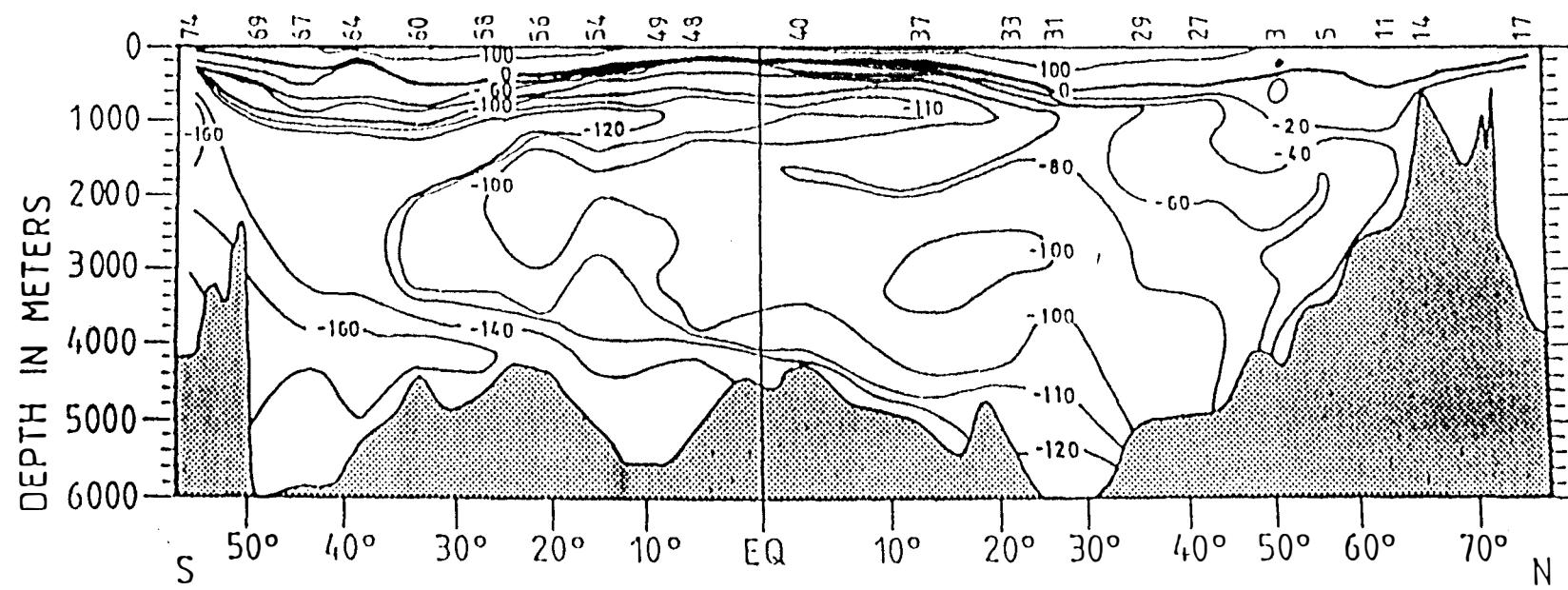


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Maier-Reimer et al. 1994

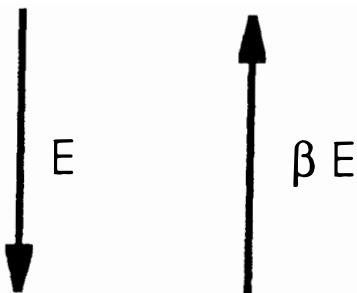
WESTERN ATLANTIC C - 14



Maier-Reimer et al 1993

Kurzwellige Einstrahlung

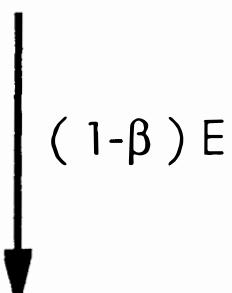
Langwellige Ausstrahlung



$$\alpha A$$

Substanzen

Substanzen



$$A$$

$$(1-\alpha) A$$

**Erdoberfläche
Ausstrahlung $A = \kappa \sigma T^4$**

Balance erfordert

$$(1-\beta) E = \alpha A$$

Daher, bei reduzierter „Durchlässigkeit“ α

$$\alpha \downarrow \Rightarrow A \uparrow \Rightarrow T \uparrow$$

Figure 1.4: Assumed nonlinear dependency of albedo ϕ upon the global mean temperature. From von Storch et al. (1998)

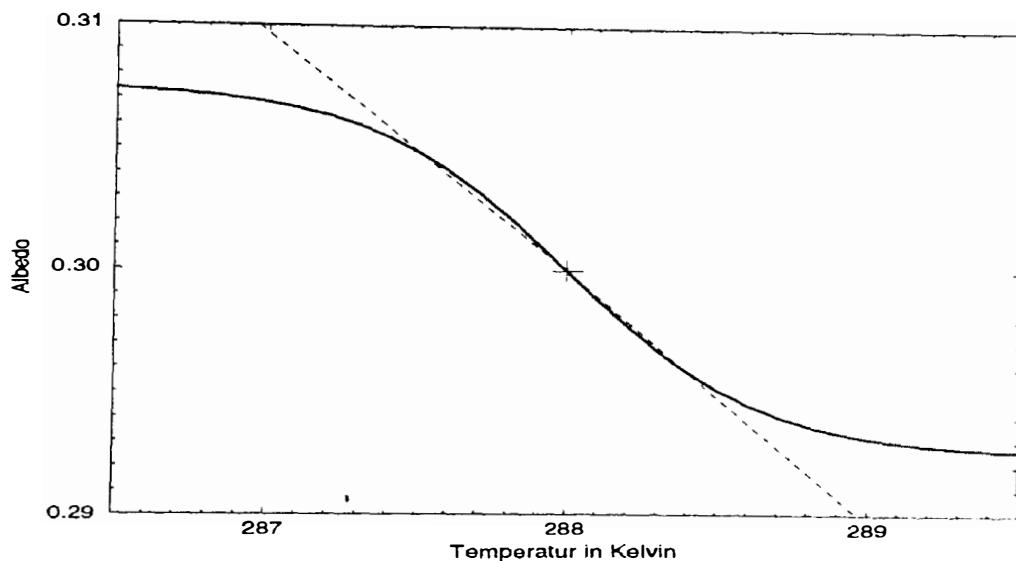


Figure 1.5: Convergence towards stable equilibrium solution of the Energy Balance Model (1.4) with temperature dependent albedo. From von Storch et al. (1998)

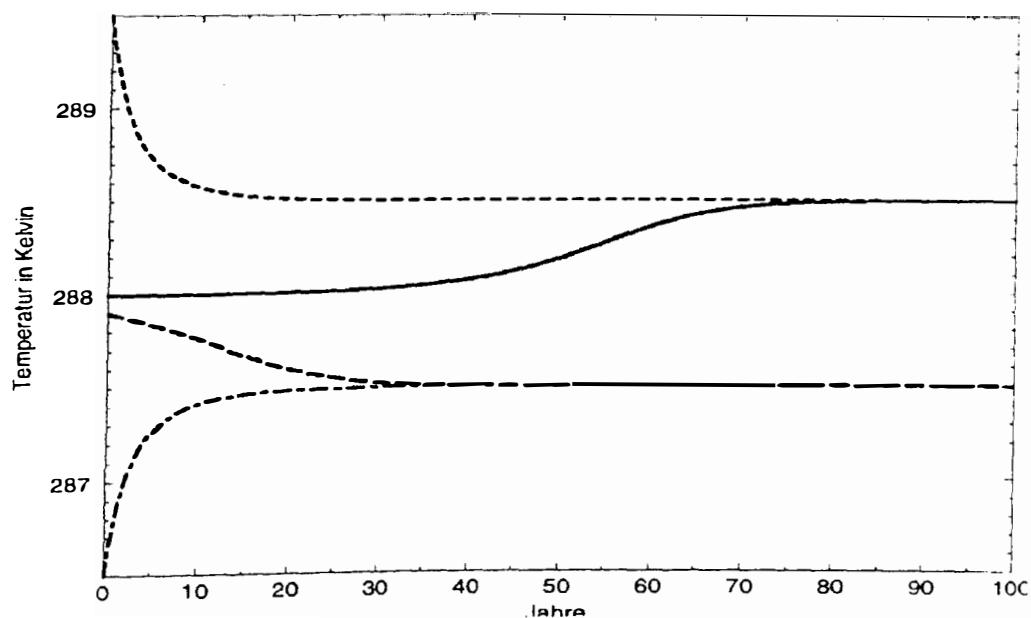
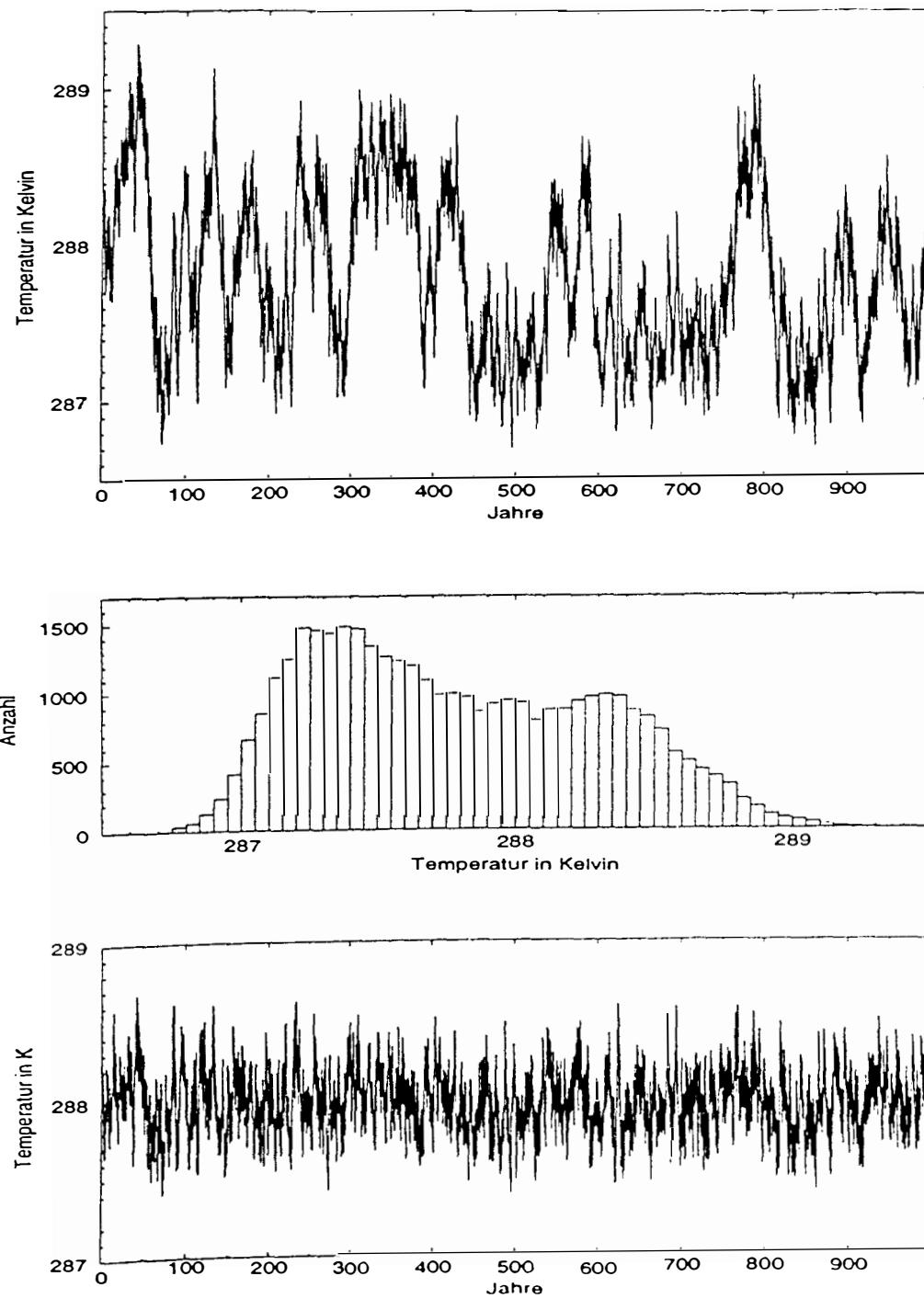


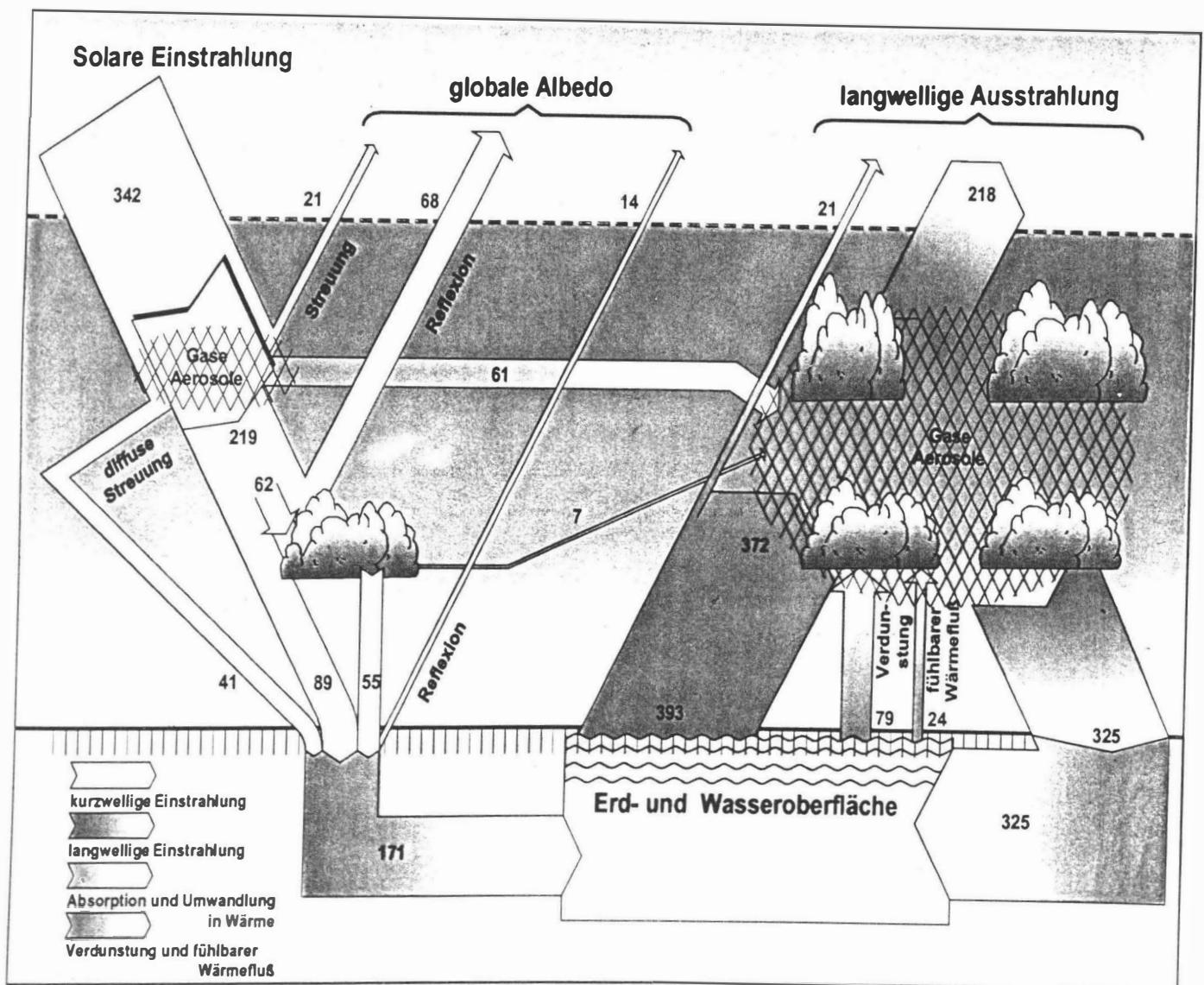
Abbildung 5.7: Oben: Langfristverhalten des EBM bei temperaturabhängiger Albedo (aus Abbildung 5.6).

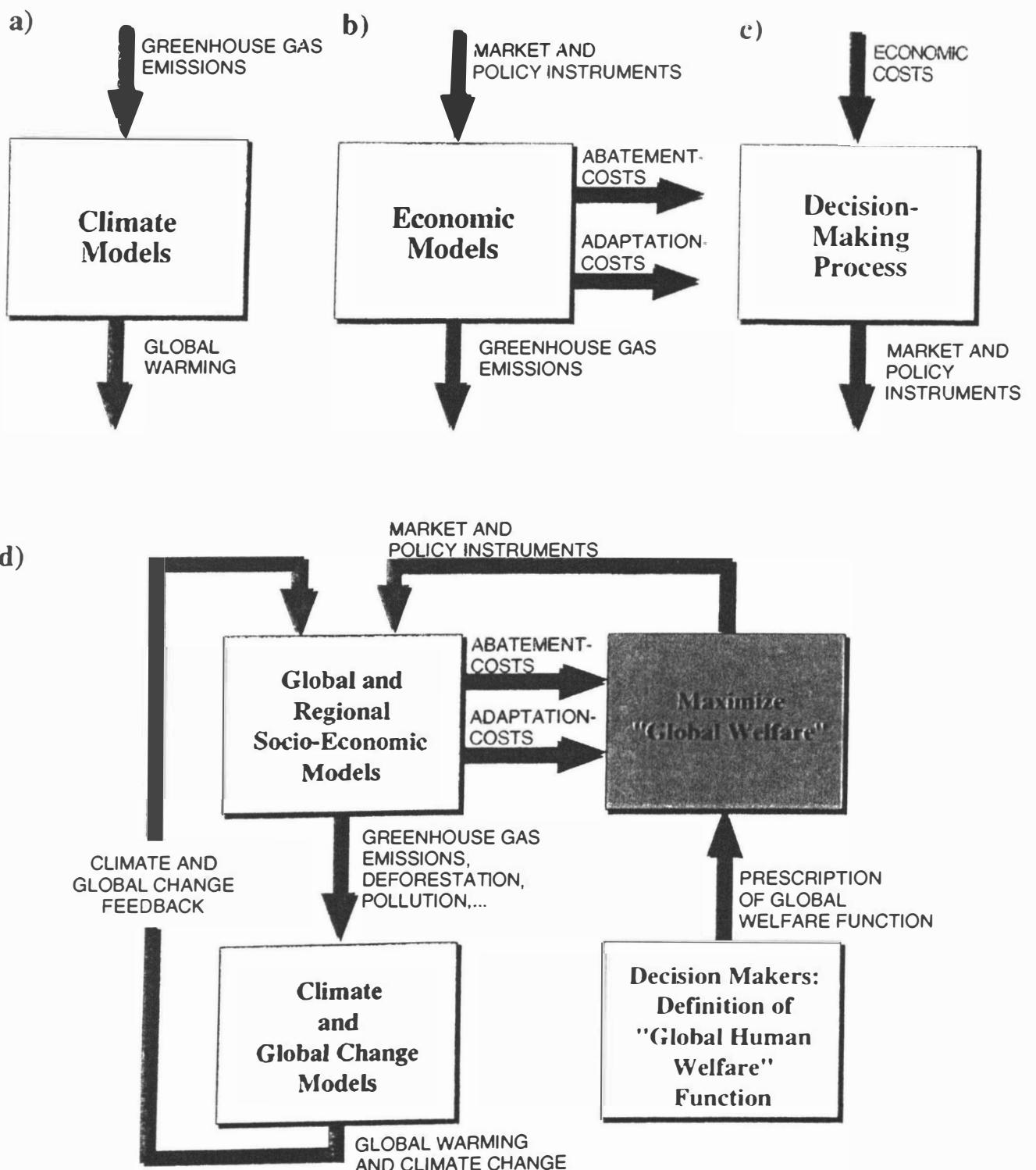
Mitte: Die Häufigkeitsverteilung des Ergebnisses.

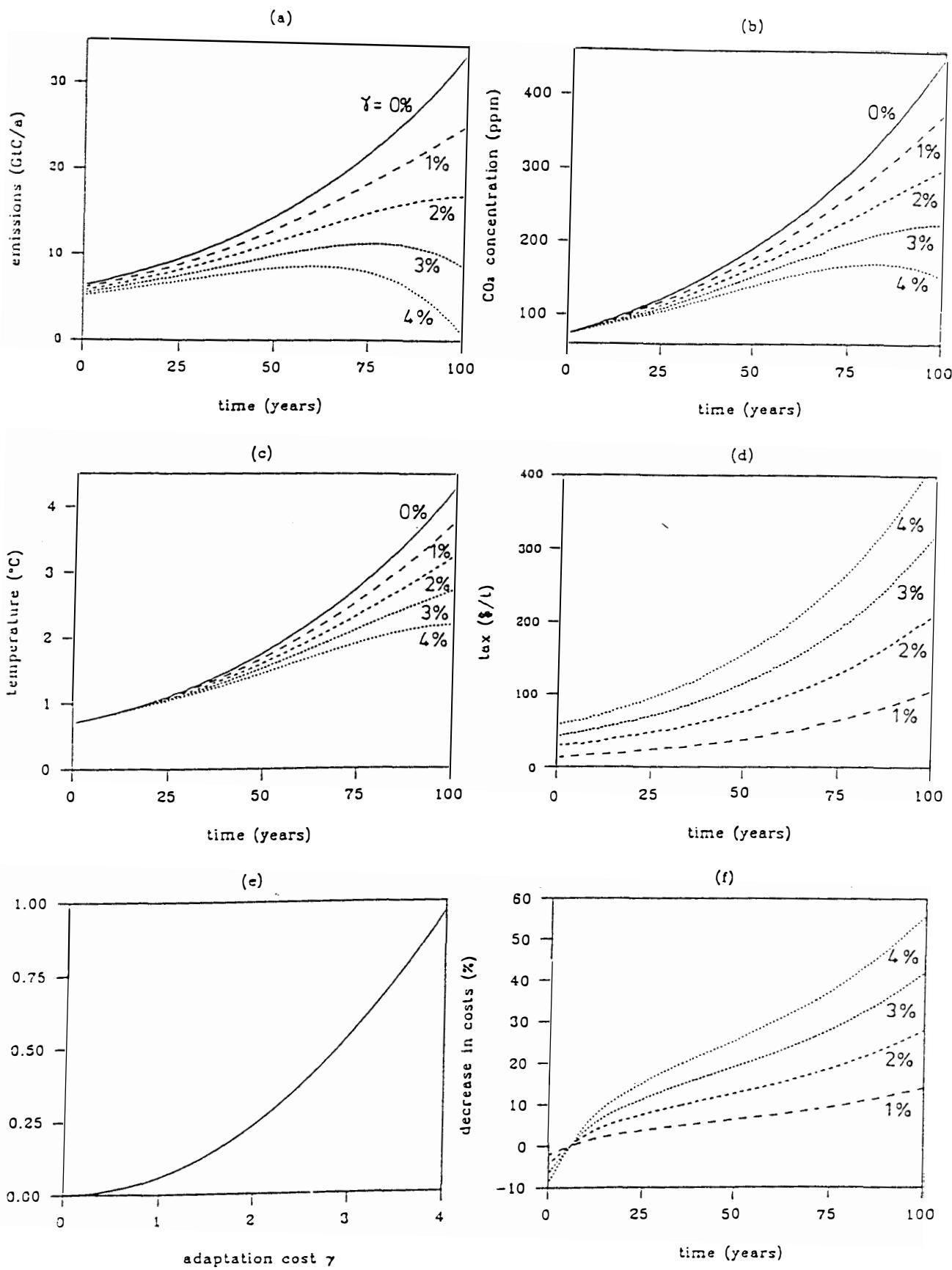
Unten: Als Vergleich das Langzeitverhalten des EBMs bei konstanter Albedo.

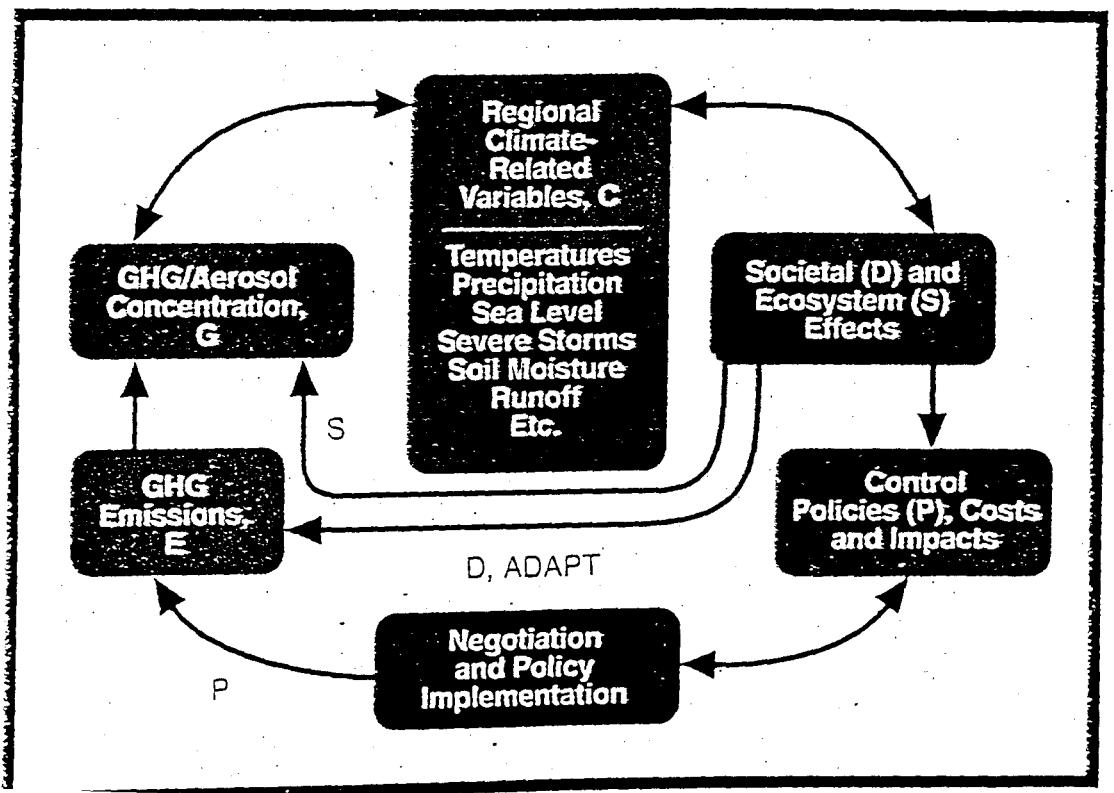


Die globale Bilanz der Strahlungsenergie, die Zahlenwerte in Watt beziehen sich auf einen m^2 Erdoberfläche. Links die kurzweligen Strahlungsflüsse, rechts rechts die langwelligen und die Wärme flüsse (fühlbar und latent)











R Roberts



„Zeitbombe Klima“ nicht im Alleingang zu entschärfen

Internationales Handeln gegen Treibhauseffekt gefordert

Lauenburg (tat). Klimazonen verschieben sich, Polkappen schmelzen, und die Luft erwärmt sich global. Über die langfristigen Folgen des Treibhauseffekts sind sich die Experten einig. Wie man sie aber vermeiden oder zumindest verzögern kann, darüber gibt es verschiedene Meinungen.

„Das Problem ist, daß es viele Interessenkonflikte gibt und daß letztlich nur internationale Maßnahmen zum Erfolg führen“, sagte Svenja Busse. Die Stipendiatin der Friedrich-Naumann-Stiftung nahm jetzt am Seminar „Zeitbombe Klima“ in der Bildungsstätte „Zündholzfabrik“ teil, und die Biologiestudentin war für die Öffentlichkeitsarbeit zuständig.

Drei Tage lang diskutierten 40 Teilnehmer aus ganz

Deutschland, überwiegend Studenten, über die Themen Ozonloch, Sommersmog und Treibhauseffekt, ermittelten Verursacher und versuchten, Regularien zur Begrenzung von Kohlendioxid (CO_2) aufzustellen.

Flugverkehr wirkt sich aus

„Das CO_2 ist im Wesentlichen dafür verantwortlich, daß die Ozonschicht immer dünner wird. Die größte Quelle sind fossile Brennstoffe“, berichtete Svenja Busse. Innerhalb dieser Gruppe kommt dem Flug-Fernverkehr eine große Bedeutung zu, besonders sehr schnellen Flugzeugen wie der Concorde oder Düsenjägern, die durch die Stratosphäre fliegen.

Über Möglichkeiten zur Minimierung von CO_2 -Emissionen sprachen Experten während einer Podiumsdiskussion. Neben der Landtagskandidatin Dr. Christel Happach-Kasan (FDP) nahmen daran auch Professor Dr. Erhard Raschke von der GKSS, Jürgen Hacker (Gesprächskreis Ökologische Marktwirtschaft), Dr. Hans-Jochen Luhmann (Wuppertal-Institut für Klima, Umwelt, Energie) und Wolfgang Podolske von der Kieler Stadtwerke AG teil.

Ihre Vorschläge reichten von verstärkter Öffentlichkeits- und Überzeugungsarbeit über monetäre Anreize beim Energieverbrauch und innovative Technologien bis zu höheren Steuern, beispielsweise auf Kraftfahrzeuge.



Diskutierten auf dem Podium (von links): Jürgen Hacker, Professor Erhard Raschke, Dr. Christel Happach-Kasan, Dr. Hans-Jochen Luhmann und Wolfgang Podolske.

Foto: Detloff

New York Times

8. July 1962

§§ 53 u. 54 UrhRG bestimmt.

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Freien Universität Berlin

—But Somebody Does Something About It

The weather, that is; many of man's activities affect it, but often without improving it.

By GEORGE H. T. KIMBLE

SOLSBERRY, Ind. — If there is one thing the farmers who live around me are agreed upon, it is that the weather is not what it used to be: it's worse. The summers, they will tell you, are stormier; the autumns wetter; the winters longer; the springs later. If there is another thing most of them are agreed upon, it is the reason for these presumed changes: "The bombs are what done it."

What evidence is there that the weather can be altered by human agency—that man can do something, whether by accident or design, to the weather? Though meteorologists may argue about how much evidence there is, they are generally agreed that it is impossible to explain what is happen-

New Haven, Conn., and Rochdale, Lancashire, have been shown to have, on the average, about 6 per cent more rain on weekdays than on Sundays.

The characteristically greater average warmth of large industrial towns compared with neighboring countrysides is likewise a product, in part, of pollution, for the greater cloudiness (itself related to the greater raininess), which the condensation nuclei induce has a "glass-house" effect—it allows the shortwave radiation (light) from the sun to pass through to the surface but prevents the longwave radiation (heat) from escaping from the surface into the free air as readily as would otherwise be the case.

BUT evidence of man-made weather

WEATHER—AND THREE MAN-MADE



THE BOMB—Its effects are not known but they may include temporary increases in the trade winds and in storminess.

I think our weather is more unpredictable now, we've got more modern satellites and all to predict the weather and I think they're worse at predictin' it now than when they didn't have them. So, your seasons don't seem to run the same. I mean, they're less predictable for patterns. . . . Like now in the spring you get real hot weather like the summer, and then when the summer comes you got weather that's like the spring, and then in the middle of winter sometimes you get weather that's almost like summer. It isn't an even pattern. *When would you say you started noticing that . . .?* I don't know, it's come on gradual.—Walt
(retired machinist)

- 1 The weather has been more variable and unpredictable recently around here.

Earth First!	Sierra Club	Public	Dry cleaners	Sawmill workers
93	52	79	73	74

Well, I like warm weather, personally, but I think it's wrong for what humans are doing to the atmosphere. *In what way?* With all the aerosols and the ozone and so forth . . . that's being projected up into the atmosphere. . . . [*If you like warm weather, why do you say the greenhouse effect would be wrong?*] Well, I think it's wrong because at the same time, we are ingesting and breathing in all these different chemicals that are being put into the atmosphere.—Susan (hospital administrator)

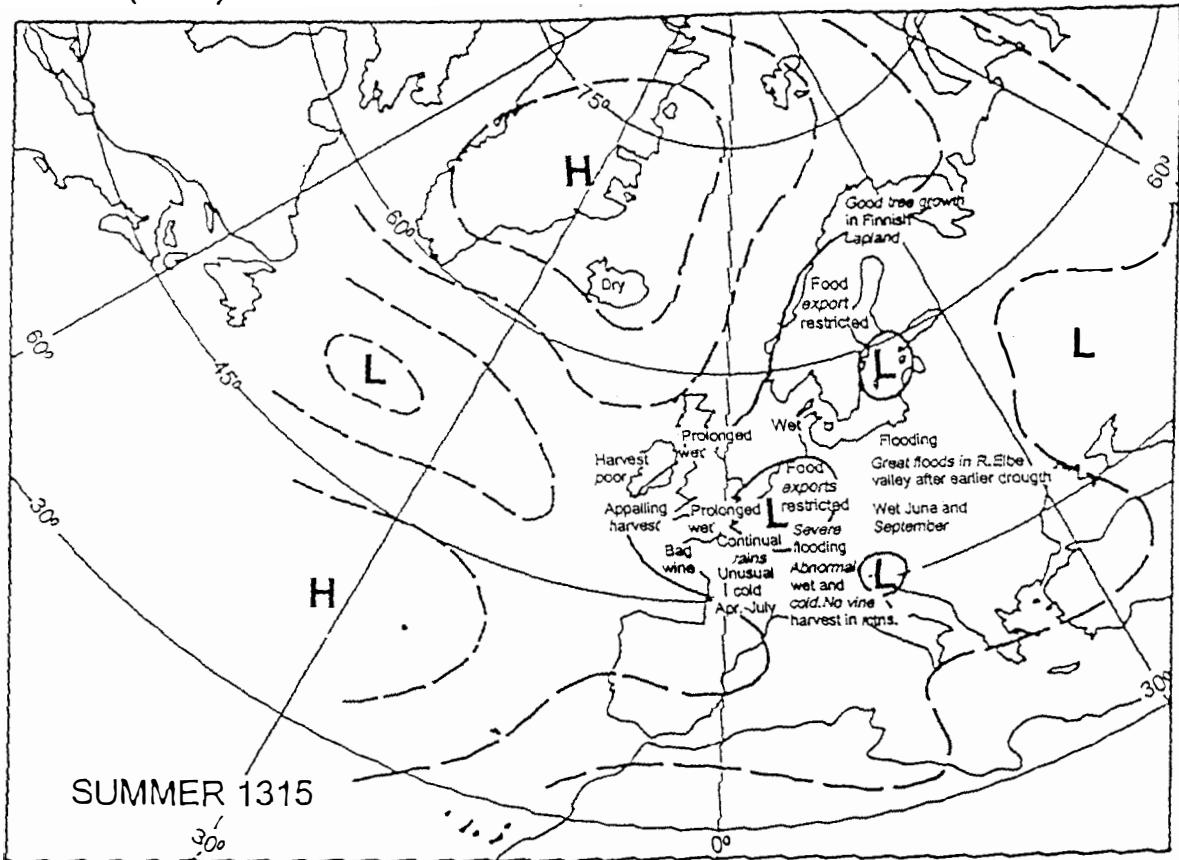
Well it has to do, I think, with the climate changing as a result of the atmosphere, atmospheric changes that are presently going, and cutting down all of our woods takes away a certain chem—the oxygen, or something, that is required for us to have good air quality, and it's kind of scary.—Tara (sales manager)

That's what scares me. *What?* When they cut all the forests down, they say, pretty soon we're not going to have any oxygen to breathe. Why do they let them do that?—Cindy (housewife)

94 If they cut all the forests down, we would soon run out of oxygen to breathe.

Earth First!	Sierra Club	Public	Dry cleaners	Sawmill workers
64	58	77	67	44

LAMB (1987)



Map of the prevailing conditions reported in the summer of 1315,
with suggested barometric pressure and wind pattern.

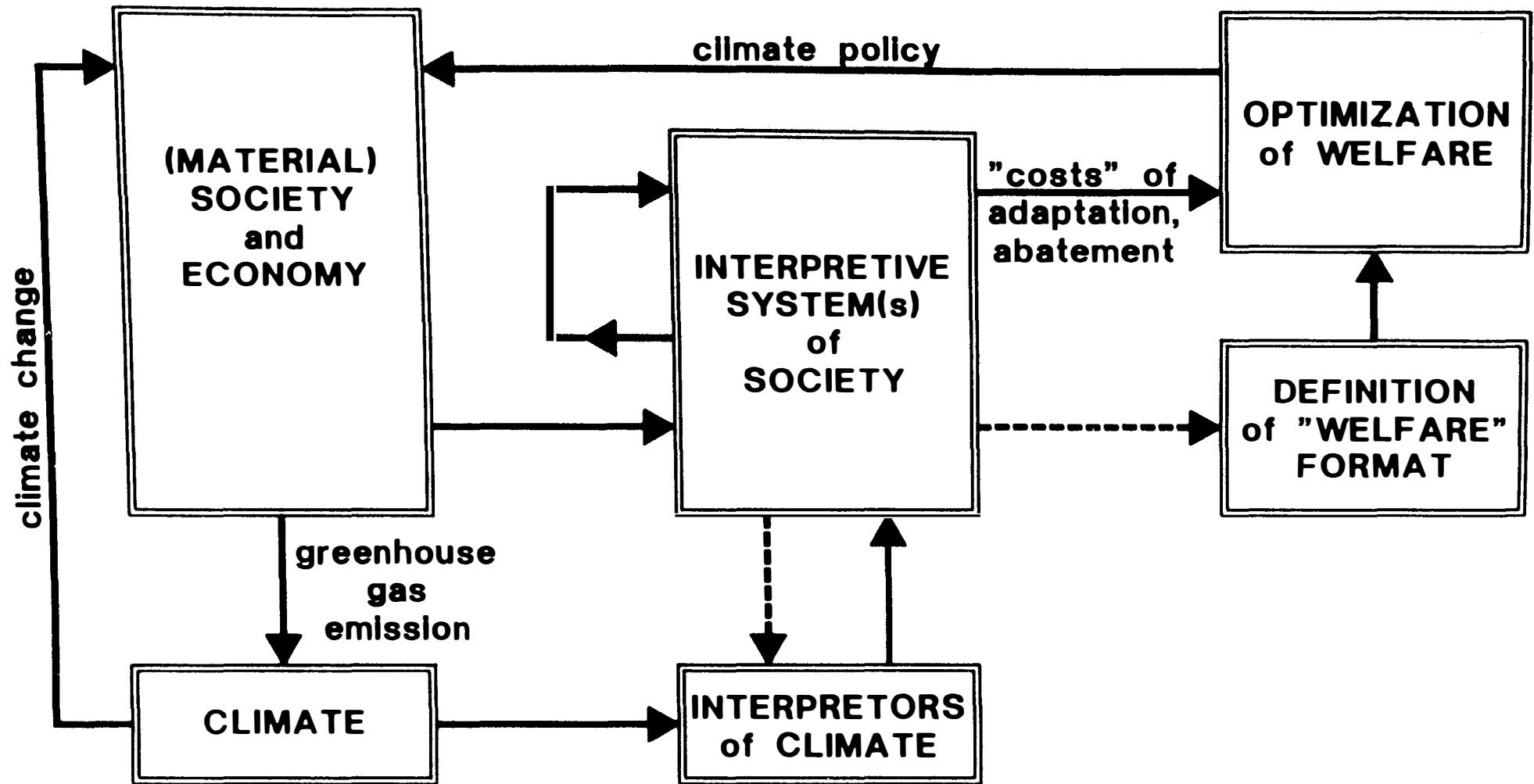
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Climate catastrophe in 1315

- Northern Europe 1315 (-1319): rainy summers, failed harvests
- High "adaption costs": famines, epidemics, ~~migh~~ mortality
- "Abatement policy" consists of various religious activities supervised by the church.
- "Abatement policy" is considerably cheaper (in units of "death" and "sin") than "adaptation policy".

Climate policy 1315

- Social authorities (church) suggest that the ongoing climate catastrophe is because of god's wrath about un-christian way of life.
- "The Archbishop of Canterbury ordered ... solemn barefooted processions ... This was in the hope of encouraging the people to atone for their sins and appease the wrath of God ..."



Folgerungen

- Gesellschaftliche Vorgänge spielen eine Rolle für die Entwicklung des Klimas.
- Gesellschaftliche Vorgänge lassen sich nur sehr bedingt in mathematische Modelle fassen: Menschen lassen sich nicht in der Art der Thermodynamik wie gleichartige von unveränderlichen Prinzipien geleitete Moleküle beschreiben.
- Naturwissenschaftler glauben bisweilen, die gesellschaftlichen Vorgänge mechanistisch in Klimamodelle einfügen zu können.
Rückkehr des Klimadeterminismus?

Folgerungen

- Klimamodelle sind weitgehend ausgereift für die realitätsnahe Beschreibung von Klimavorgängen auf der Zeitskala von Tagen bis zu wenigen Jahren im Bereich des heutigen Klimas.
- Klimamodelle können verkomplettiert werden durch Hinzufügung anderer naturwissenschaftlicher Komponenten.

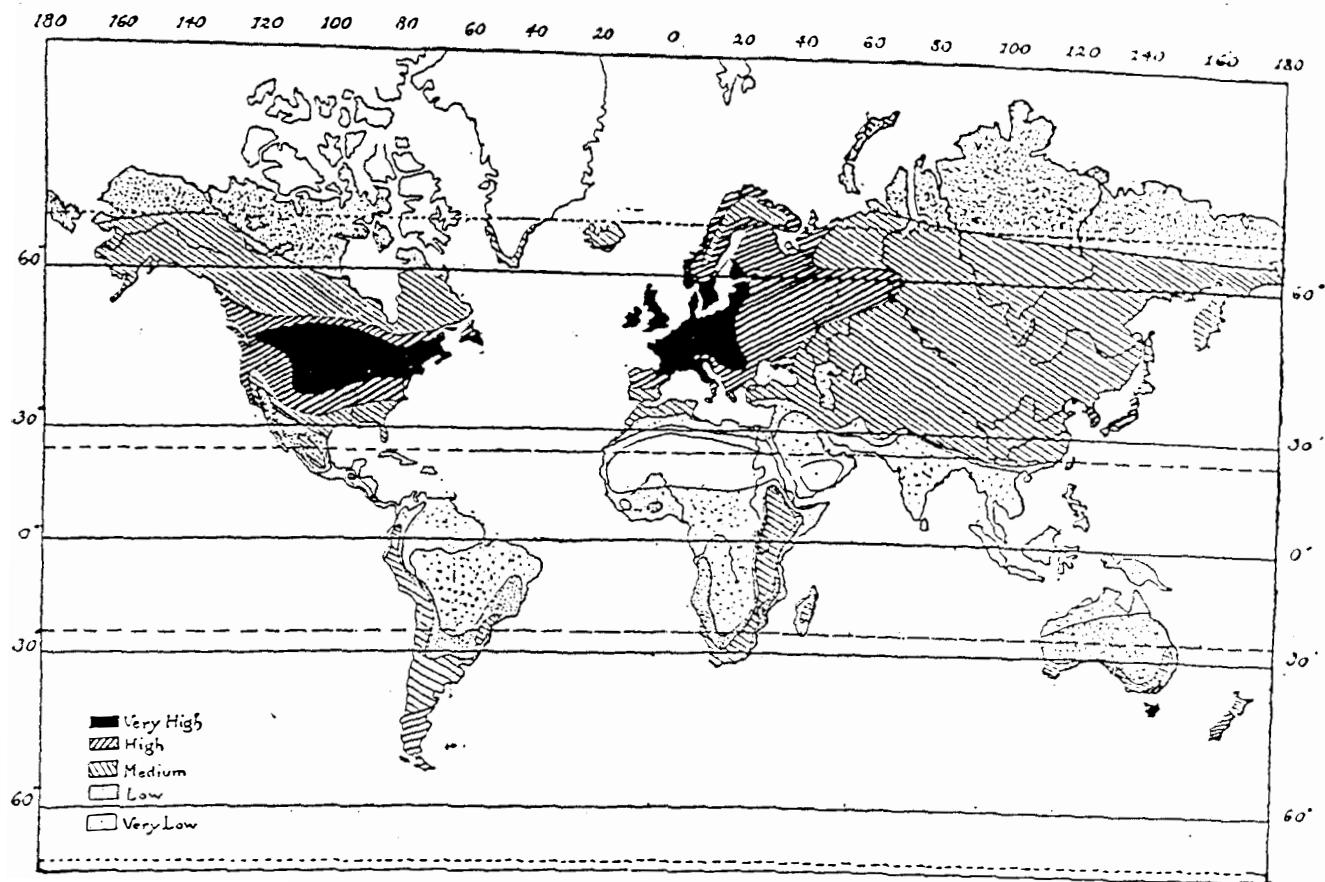


Figure 43. The Distribution of Human Health and Energy on the Basis of Climate

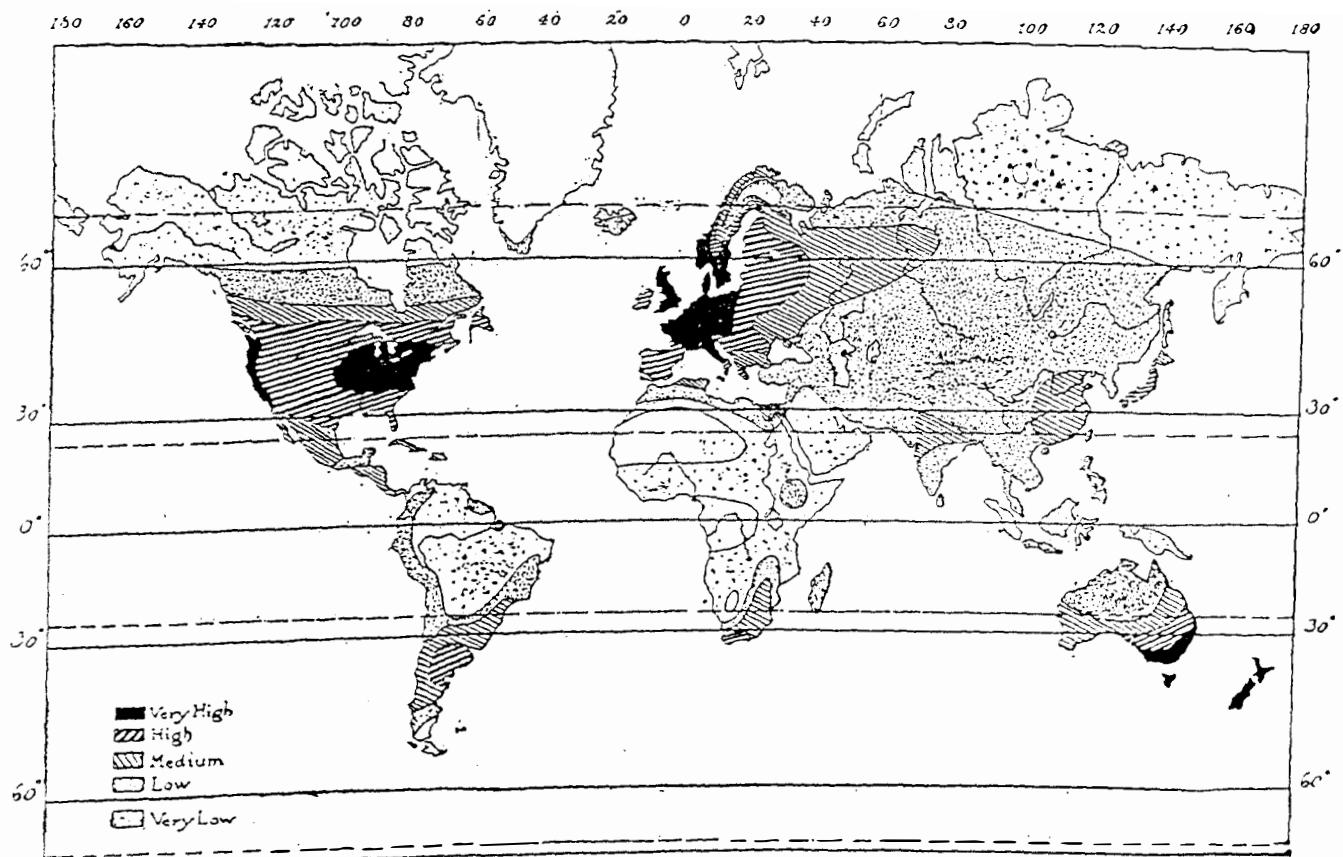


Figure 44. The Distribution of Civilization

Viewpoint

Climate, liberalism and intolerance

Several writers have remarked on apparent correlations between the character of the people of a region and the climate prevailing there; thus Manley (1971) associated the flexible British character with the changeable British climate, whilst Miller

Conversely, intolerant acts have often been committed by people from areas in mid-latitudes where seasonal temperature extremes are large, as in areas with continental climate. In the 1930s, fascism took over in Spain, Germany, Italy and Austria; all are continental countries with TD values generally averaging about 20 deg C (one exception is southern Italy, which has a TD of about 15 deg C, but early support for the fascist Mussolini was said to be weak). Ency-

Many of the states of the USA which retain capital punishment have TD values of over 20 deg C, which is high compared with most other 'western' nations.

It may never be possible to prove absolutely that a mild climate in mid-latitudes helps to foster a tolerant society or that an extreme climate may predispose people towards intolerance. However, the historical record is highly suggestive and if this is recognised it could help to identify potential problem areas in the field of human relations so that timely action can be taken to mitigate threats to peace. There must be no attempt to justify intolerance on climatic grounds but only to note that certain climates may be associated with a predisposition to such conduct. Perhaps the absence of seasonal extremes helps to foster a relaxed attitude because there is no need to make elaborate plans to cope with the rigours of a cold winter and/or a very hot summer. However, where TD is large, the pace of life is driven by the seasons, enforcing the discipline of timely preparation for the extremes; here, less relaxed mental attitudes may develop.

If the weather gets a lot warmer, do you think it would be good, bad, or neutral?
I think it would be bad; I think it would be terrible. *Why?* Well, I think people react differently in warm weather than when it's cooler. I think it has an effect on attitudes—behavior. . . . I mean in the prison system especially, where the people are just, you know, stuck in there, and they've got to let off steam. So, sure. *So you think in prison it makes people more violent?* Sure, but outside the prisons, too, 'cause I even see it at work; you know, when the weather is extremely warm, people tend to be, you know, a little hot tempered. I think, you know, their blood boils. And when the blood boils in the body, it goes to the head, and next thing you know, there's, you know, an explosion. . . . I've seen them react that way.—Paige (manufacturing worker)

Methoden zur Beschreibung dominanter Muster in geophysikalischen Vektorzeitreihen

Hans von Storch
GKSS Forschungszentrum

Basierend auf Kapitel 13, “Spatial patterns: EOFs and CCA”
in: von Storch, H., and A. Navarra (Eds.), 1995: Analysis of Climate Variability:
Applications of Statistical Techniques, Springer Verlag, (ISBN 3-540-58918-X)

Signal and Noise Subspace, Guess Patterns

The separation of the full phase space into a “signal” subspace, spanned by a few patterns \vec{p}^k and a “noise” subspace may be formally written as

$$\vec{\mathbf{X}}_t = \sum_{k=1}^K \alpha_k(t) \vec{p}^k + \vec{n}_t$$

with t representing time. The K “guess patterns” \vec{p}^k and time coefficients $\alpha_k(t)$ are supposed to describe the dynamics in the signal subspace, and the vector \vec{n}_t represents the “noise subspace”.

The truncated vector of state

$$\vec{\mathbf{X}}_t^S = \sum_{k=1}^K \alpha_k(t) \vec{p}^k$$

is the projection of the full vector of state on the signal subspace. The residual vector

$$\vec{n}_t = \vec{\mathbf{X}}_t - \vec{\mathbf{X}}_t^S$$

represents the contribution from the noise subspace.

Expansion Coefficients

- The “expansion coefficients” $\vec{\alpha} = (\alpha_1, \alpha_2 \dots \alpha_K)^T$ are determined as those numbers which minimize

$$\epsilon(\vec{\alpha}) = \langle \vec{\mathbf{X}} - \sum_k \alpha_k \vec{p}^k, \vec{\mathbf{X}} - \sum_k \alpha_k \vec{p}^k \rangle$$

with the “dot product” $\langle \vec{a}, \vec{b} \rangle = \sum_j a_j b_j$. The optimal vector of expansion coefficients is obtained as a zero of the first derivative of ϵ :

$$\sum_{i=1}^K \vec{p}^{kT} \vec{p}^i \alpha_i = \vec{p}^{kT} \vec{\mathbf{X}}$$

After introduction of the notation $\mathcal{A} = (\vec{a} | \vec{b} \dots)$ for a matrix \mathcal{A} with the first column given by the vector \vec{a} and the second column by a vector \vec{b} ,

$$\mathcal{P}\vec{\alpha} = (\vec{p}^1 | \dots | \vec{p}^K)^T \vec{\mathbf{X}}$$

with the symmetric $K \times K$ -matrix $\mathcal{P} = (\vec{p}^{kT} \vec{p}^i)$. In all but pathological cases the matrix \mathcal{P} will be invertible such that a unique solution exists:

$$\vec{\alpha} = \mathcal{P}^{-1} (\vec{p}^1 | \dots | \vec{p}^K)^T \vec{\mathbf{X}}$$

Finally, if we define K vectors $\vec{p}_A^1 \dots \vec{p}_A^K$ so that

$$(\vec{p}_A^1 | \dots | \vec{p}_A^K) = (\vec{p}^1 | \dots | \vec{p}^K) \mathcal{P}^{-1}$$

Then the k -th expansion coefficient α_k is given as the dot product of the vector of state $\vec{\mathbf{X}}$ and the “adjoint pattern” \vec{p}_A^k :

$$\alpha_k = \langle \vec{p}_A^k, \vec{\mathbf{X}} \rangle$$

- In some cases, and in particular in case of EOFs, the patterns \vec{p}^k are orthogonal such that \mathcal{P} is the identity matrix and $\vec{p}^k = \vec{p}_A^k$. In this case

$$\alpha_k = \langle \vec{p}^k, \vec{\mathbf{X}} \rangle$$

A Measure of Skill: Explained Variance

- A measure to quantify the importance of a set of patterns $\{\vec{p}^k\}$ is the “amount of explained variance”, or, more precisely, the “proportion of variance accounted for by the $\{\vec{p}^k\}$ ”:

$$\eta = \frac{\text{VAR}(\vec{\mathbf{X}}) - \text{VAR}(\vec{\mathbf{X}} - \sum_k \alpha_k \vec{p}^k)}{\text{VAR}(\vec{\mathbf{X}})}$$

- The numerical value of the explained variance is bounded by $-\infty < \eta \leq 1$.
 If $\eta = 0$ then $\text{VAR}(\vec{\mathbf{X}} - \sum_k \alpha_k \vec{p}^k) = \text{VAR}(\vec{\mathbf{X}})$ and the representation of $\vec{\mathbf{X}}$ by the patterns is useless since the same result would have been obtained by any patterns and $\alpha_k = 0$.
 $\eta = 1$ implies $\text{VAR}(\vec{\mathbf{X}} - \sum_k \alpha_k \vec{p}^k) = 0$ and thus a perfect representation of $\vec{\mathbf{X}}$ by the guess patterns \vec{p}^k .
- The amount of explained variance can also be defined locally for each component j :

$$\eta(j) = 1 - \frac{\text{VAR}(\mathbf{X}_j - \sum_k \alpha_k p_j^k)}{\text{VAR}(\mathbf{X}_j)}$$

If the considered random vector $\vec{\mathbf{X}}$ can be displayed as a map then also the amount of explained variance η can be visualized as a map.

Example: Temperature in the Mediterranean Sea

The output of a 9-year run of an OGCM, forced by monthly mean atmospheric conditions for the years 1980 to 1988, was decomposed

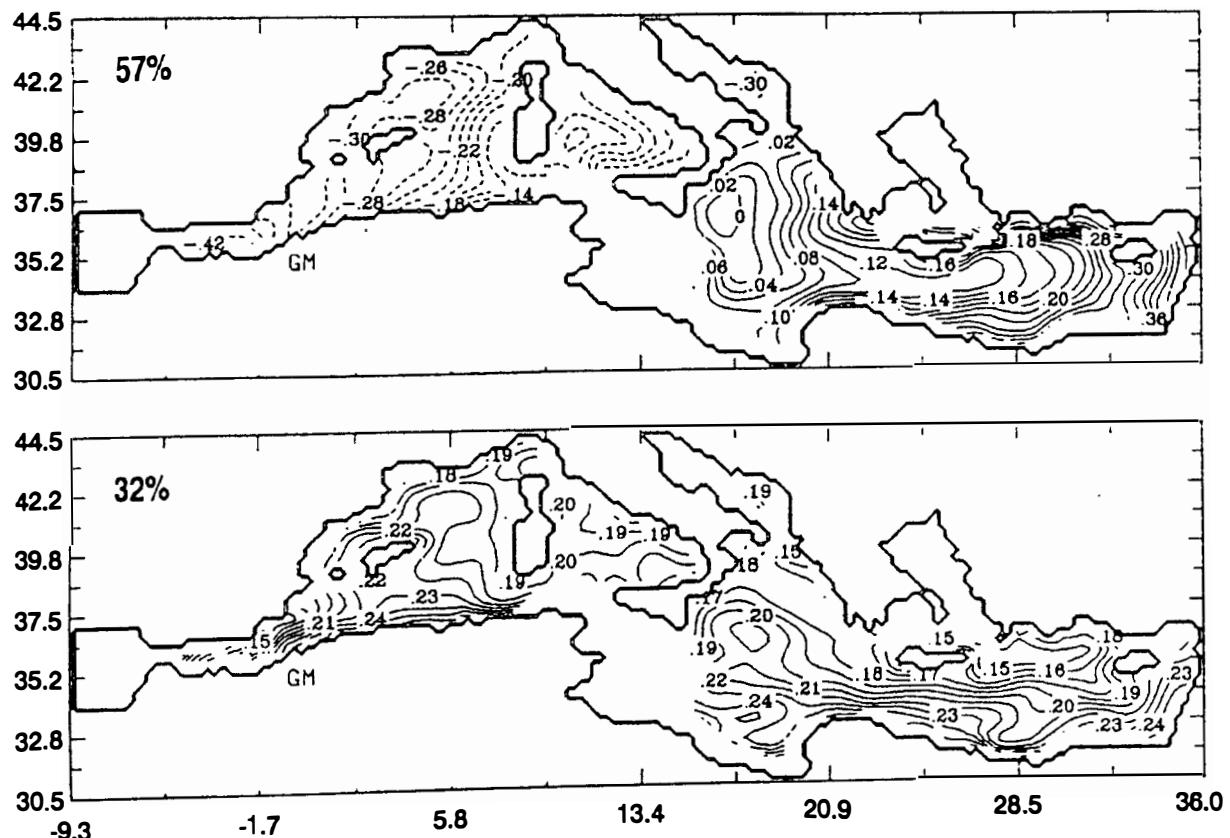
$$\vec{T}(\vec{r}, z, t) = \sum_k \alpha_k(z, t) \vec{p}_r^k$$

with \vec{r} representing the horizontal coordinates,
 z the vertical coordinate and
 t the time.

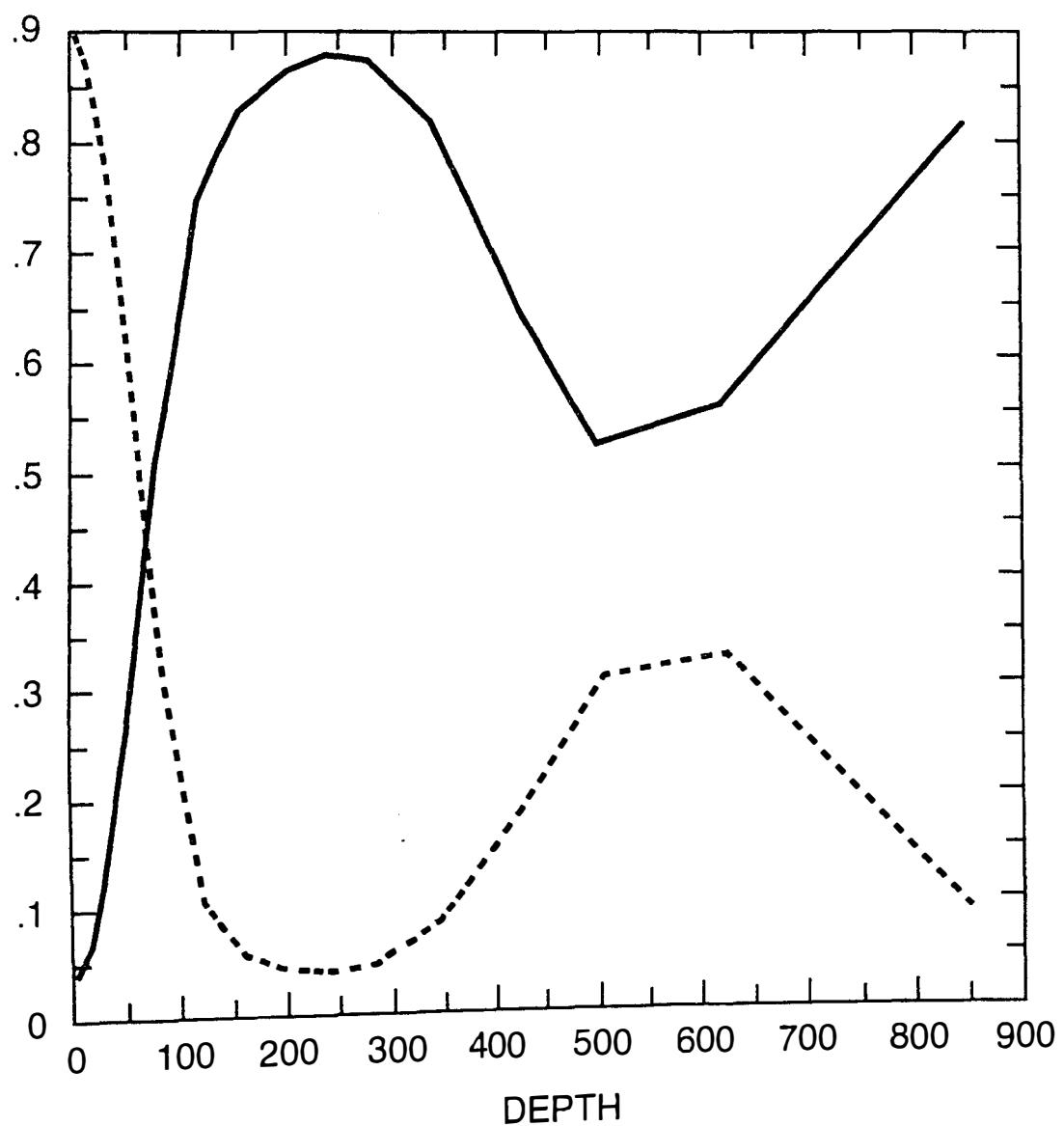
The temperature field is given on a (\vec{r}, z) -grid with better resolution in the upper levels.

The coefficients α_k depend on depth and time. The orthogonal patterns \vec{p}^k describe the horizontal distribution and are independent of the depth and time.

First two characteristic horizontal distributions \vec{p}_r^k of normalized temperature in the Mediterranean Sea, as inferred from the output of a 9-year run with a numerical ocean model. (From Korres and Pinardi).



Vertical distribution of the percentage of the 2nd moment of the normalized temperature accounted for by the first and second patterns.



Specification of Guess Patterns

- “**Principal Interaction Patterns**” (**PIP**) are implicitly defined such that their coefficients $\alpha_k(t)$ approximate certain dynamical equations, which feature unknown parameters.
- “**Principal Oscillation Patterns**” (**POPs**) are a simplified version of PIPs and model linear dynamics.
- **Expected signals**, such as the atmospheric response to enhanced greenhouse gas concentrations or to anomalous sea-surface temperature conditions are often available from experiments with general circulation models. Also patterns “predicted” by simplified dynamical theory (for instance, linear barotropic equations) are in use.
- **Orthogonal functions** such as trigonometric functions or spherical harmonics are often used. In all “spectral” atmospheric general circulation models the horizontal fields are expanded with spherical harmonics as guess patterns.
- **Empirical Orthogonal Functions (EOFs) and Canonical Correlation Patterns (CCPs)** will be discussed in this presentation.
Offsprings are Extended EOFs (EEOFs) and Complex EOFs (CE-OFs), Singular Spectrum Analysis (SSA) and Multichannel Singular SPectrum Analysis (MSSA).
- “**Wavelet**” **analysis** projects a time series on a set of patterns, which are controlled by a location and a dispersion parameter.

Rotation

- **The name “Rotated EOFs”** is misleading as it indicates that the “rotation” would exploit properties special to the EOFs. This is not the case.
- **The general concept of “rotation”** is to replace the patterns \vec{p}^k by “nicer” patterns \vec{p}_R^k :

$$\sum_{k=1}^K \alpha_k \vec{p}^k = \sum_{k=1}^K \alpha_k^R \vec{p}_R^k$$

The patterns \vec{p}_R^k are determined such that they maximize a certain functional of “simplicity” and that they span the same space as the original set of vectors $\{\vec{p}^k\}$. Constraints like unit length

$$\vec{p}_R^k{}^T \vec{p}^k = 1$$

and, sometimes, orthogonality

$$\vec{p}_R^k{}^T \vec{p}^i = 0$$

are invoked.

Empirical Orthogonal Functions

Theorem:

The first K eigenvectors \vec{p}^k , for any $K \leq m$, of the covariance matrix $\Sigma = E(\vec{\mathbf{X}}\vec{\mathbf{X}}^T)$ of the m -variate random vector $\vec{\mathbf{X}}$ form a set of pairwise orthogonal patterns. They minimize the variance

$$\epsilon_K = E\left(\left(\vec{\mathbf{X}} - \sum_{k=1}^K \alpha_k \vec{p}^k\right)^2\right) = \text{VAR}(\vec{\mathbf{X}}) - \sum_{k=1}^K \lambda_k \quad (1)$$

with $\alpha_k = \vec{\mathbf{X}}^T \vec{p}^k$ and $\vec{p}^{kT} \vec{p}^k = 1$. The patterns are named “Empirical Orthogonal Functions”.

- EOFs are geometricaly orthogonal and the “EOF coefficients” α_i are *statistically independent*, or, more correctly, share zero-correlations among each other:

$$E(\alpha_i \alpha_k) = \vec{p}^{iT} E(\vec{\mathbf{X}}\vec{\mathbf{X}}^T) \vec{p}^k = \vec{p}^{iT} \Sigma \vec{p}^k = \lambda_k \vec{p}^{iT} \vec{p}^k = \lambda_k \delta_{k,i}$$

Also

$$\text{VAR}(\alpha_k) = \lambda_k$$

- For the explained variance, we find

$$\eta_{\{1\dots K\}} = 1 - \frac{\epsilon_K}{\text{VAR}(\vec{\mathbf{X}})} = 1 - \frac{\sum_{k=K+1}^m \lambda_k}{\sum_{k=1}^m \lambda_k} = \frac{\sum_{k=1}^K \lambda_k}{\sum_{k=1}^M \lambda_k}$$

What EOFs Are Not Designed for . . .

- EOFs are constructed to represent in an optimal manner *variance* and *covariance* (in the sense of *joint variance*), not *physical connections* or *maximum correlation*. Therefore they are excellent tools to *compress* data into a few variance-wise significant components.
- Sometimes people expect a description of the “coherent structures”. This goal can be achieved only when the data are normalized to variance one, i.e., if the correlation matrix instead of the covariance matrix is considered.
- Another expectation is that EOFs would tell us something about the structure of an underlying continuous field from which the data vector $\vec{\mathbf{X}}$ is sampled. This is not so. The details of spatial / temporal sampling matter.
- Also often EOFs are thought to represent modes of “natural” or “forced” variability. This view is often adequate for the first EOF but mostly inadequate for all other EOFs.

Estimation of EOFs

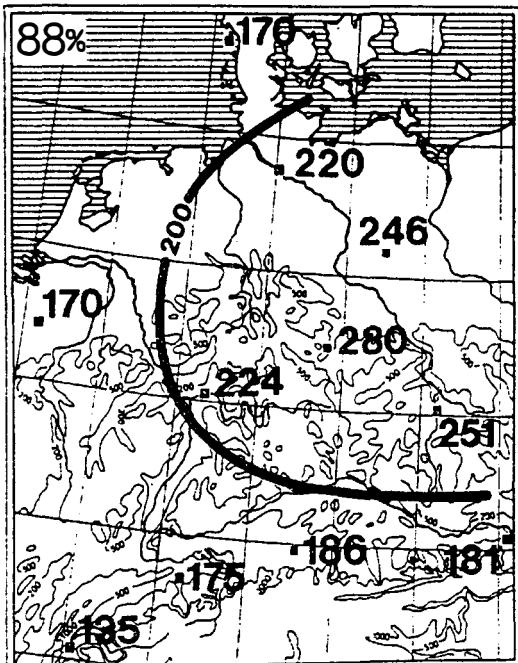
- Usually done by centering data, and replacing expectation operators by sums.
- Low-index EOFs and eigenvalues are usually better estimated than high-indexed EOFs and eigenvalues.
- “**Significance tests**” are designed for detecting degeneracy of eigenvalues, not for assessing the quality of the estimation of the eigenvectors.

Example: Central European Temperature in Winter

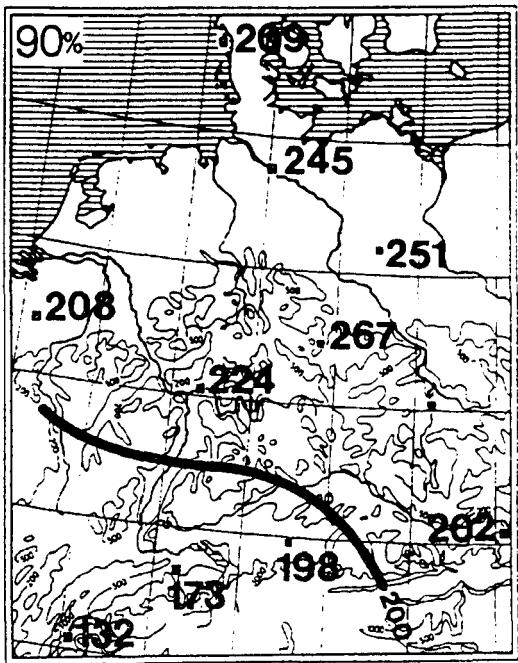
- Werner, P.C. and H. von Storch, 1993: Interannual variability of Central European mean temperature in January/February and its relation to the large-scale circulation. - Clim Res. 3, 195-207
- Winter mean temperature anomalies (i.e., deviations from the overall winter mean) at eleven Central European stations.
- Homogeneous time series were available for eighty winters from 1901 to 1980.
- For a better display of the results sometimes a different normalization is convenient, namely $\alpha_k \vec{p}^k = (\alpha_k / \sqrt{\lambda_k}) \times (\vec{p}^k \sqrt{\lambda_k}) = \alpha'_k \vec{p}'^k$. In this normalization the coefficient time series has variance one for all indices k and the relative strength of the signal is in the patterns \vec{p}' . A typical coefficient is $\alpha' = 1$ so that the typical reconstructed signal is \vec{p}' .
- EOFs were calculated for 1901-40 and 1941-80 separately.

Mean temperature in JF (10^{-2} K)

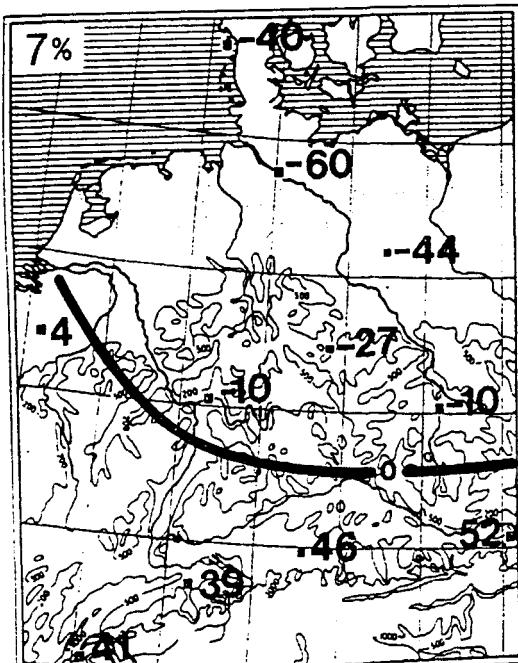
1901-40



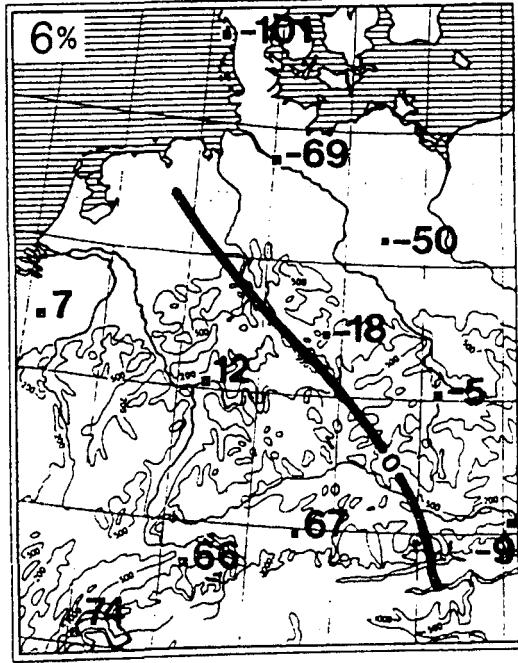
1941 - 80



1.EOF

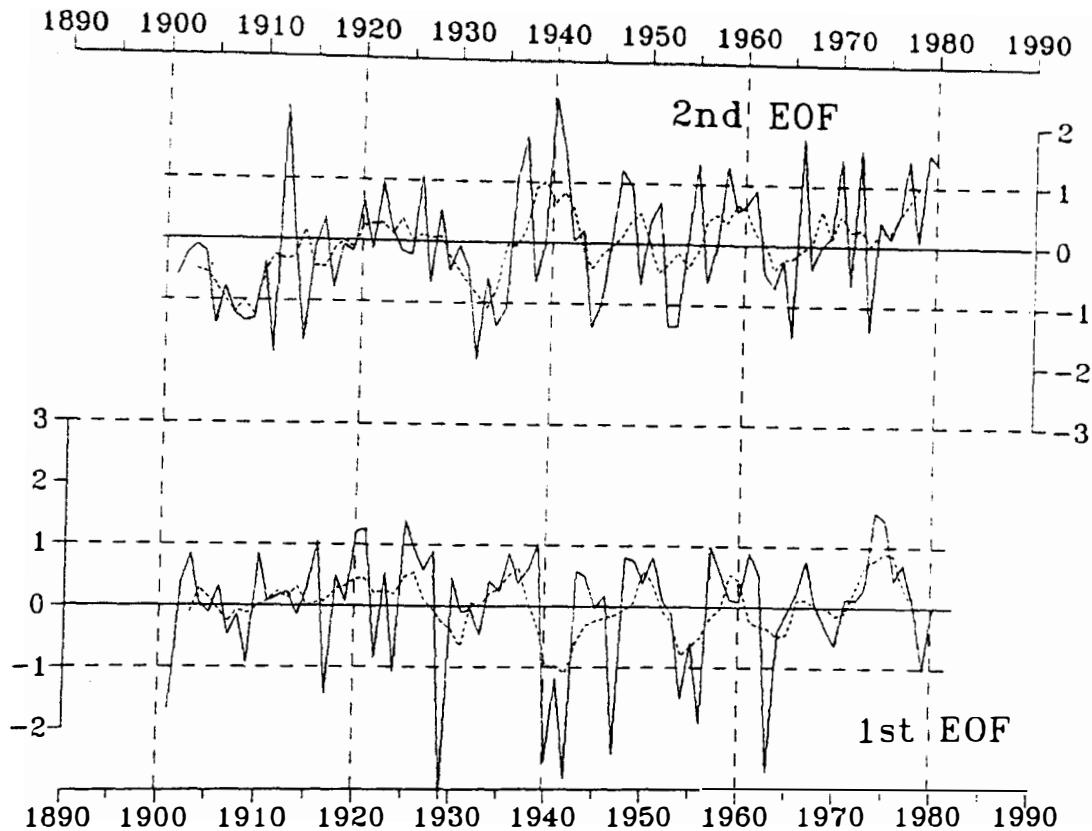


1.EOF

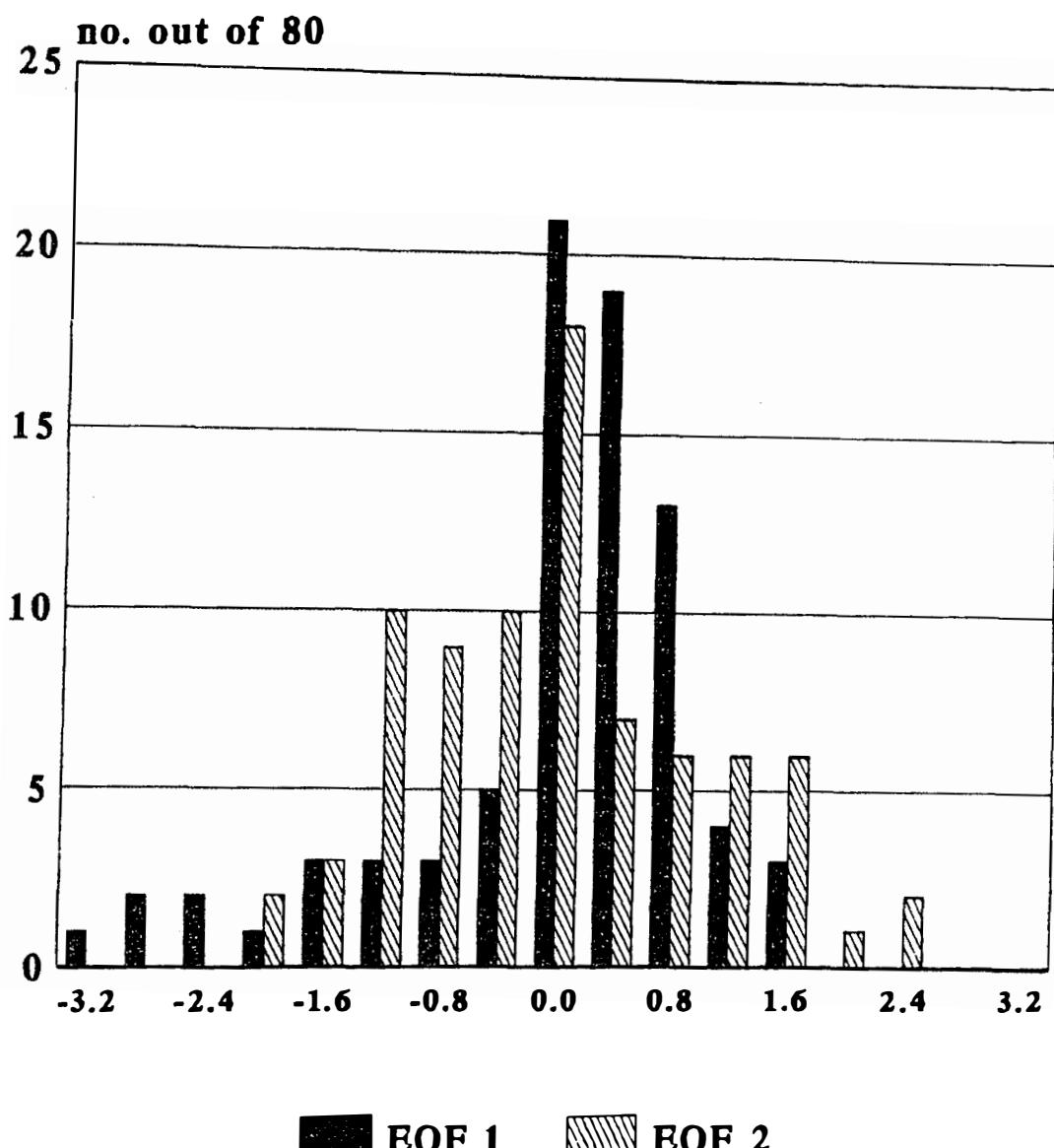


2.EOF

First two EOFs of January-February mean temperature at 11 Central European stations derived from the winters 1901-40 (left) and from the winters 1941-80 (right). Units: 10^{-2}°C .



EOF coefficient time series $\alpha_1(t)$ and $\alpha_2(t)$ of the first two EOFs of winter mean temperature at 11 Central European stations. Note that the time series have been normalized to one so that the information about the strength of the variation is carried by the patterns.



Frequency distribution of the EOF coefficient time series

Canonical Correlation Analysis

In the Canonical Correlation Analysis [CCA, proposed by Hotelling (1936) a pair of two simultaneously observed vectors $\vec{\mathbf{X}}$ and $\vec{\mathbf{Y}}$:

$$\vec{\mathbf{X}}_t = \sum_{k=1}^K \alpha_k^X(t) \vec{p}_X^k \quad \text{and} \quad \vec{\mathbf{Y}}_t = \sum_{k=1}^K \alpha_k^Y(t) \vec{p}_Y^k$$

with the same number K .

The dimensions m_X and m_Y of the vectors \vec{p}_X^k and \vec{p}_Y^k will in general be different. The expansion is done in such a manner that

1. The correlations

- between α_k^X and α_l^X
- between α_k^Y and α_l^Y
- between α_k^X and α_l^Y

are zero for all $k \neq l$.

2. The correlation between α_1^X and α_1^Y is maximum.

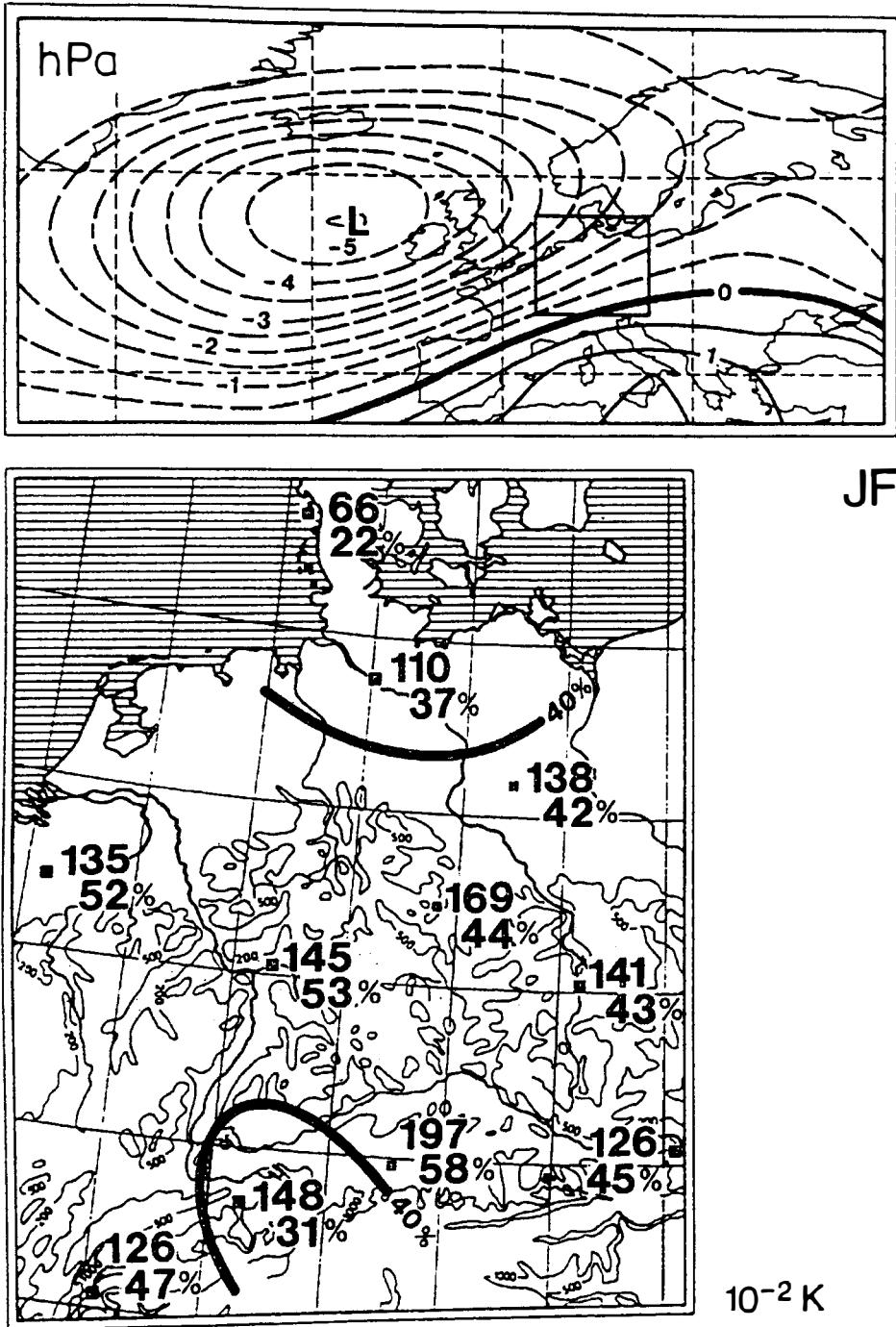
3. The correlation between α_k^X and α_k^Y is the maximum under the constraints of 1).

Pre-Processing

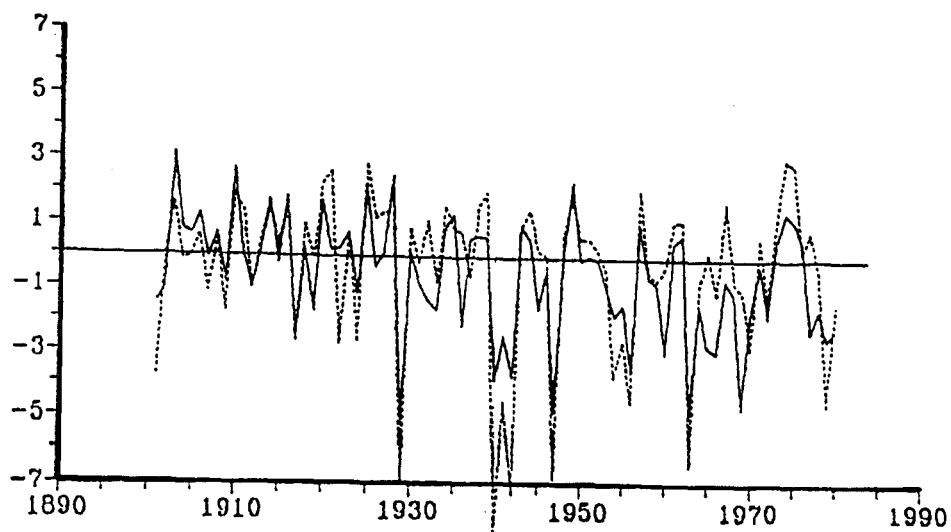
- A plain CCA of vector series with many components and few samples results in sometimes overestimations of the canonical correlations.
- To overcome this problem, it is often advisable to pre-process the data by projecting $\vec{\mathbf{X}}$ and $\vec{\mathbf{Y}}$ on their first few EOFs.
- In climate research, practically all CCAs are done with an a priori EOF filtering.

Example: Central European Temperature

- Simultaneous analysis of the large-scale circulation, as given by the seasonal mean sea-level air-pressure (SLP) anomaly over the North Atlantic and Europe, and the Central European temperature given at 11 locations.
- The objective of this exercise was to determine to what extent the regional temperature is controlled by large-scale circulation anomalies.
- With first 40 years of the full 1901-1980 data set the CCA was done.
- The data were first projected on EOFs. The number of EOFs retained was determined in such a way that an increase by one EOF would change the canonical correlations only little.
- With the full data set, from 1901-80 the correlation of the coefficients α_1^T and α_1^{SLP} is recalculated and found to be only 0.64 compared to 0.70 derived from the “fitting” interval 1901-40.



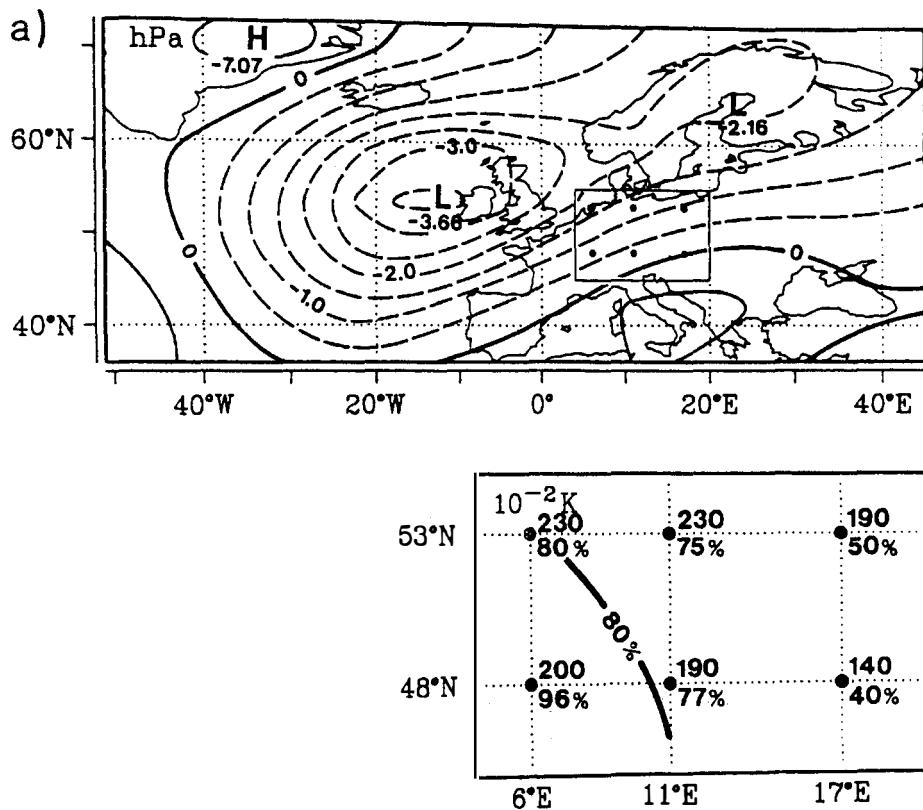
First pair of canonical correlation patterns \vec{p}_T^1 of Central European temperature (upper numbers; in $10^{-2} \text{ }^\circ\text{C}$) and \vec{p}_{SLP}^1 of North Atlantic/European sea-level air-pressure (in hPa) in winter.
 The patterns are normalized such that the coefficients α_1^T and α_1^{SLP} have variance one.
 The percentage numbers are the amount η of temperature variance accounted for by the \vec{p}_T^1 -pattern.



Time series of winter mean temperature in Hamburg (deviations from the 1901-40 mean) as derived from in-situ observations (dotted) and as derived indirectly from large-scale SLP patterns by means of CC patterns (solid line).

Application: Verification of GCM

- The output of a climate GCM has been analysed whether it reproduces the connection represented by the CCA-pairs. The regional temperature from a GCM output is given at grid points. Therefore the 11 Central European stations are replaced by 6 grid points.
- The first pair of CC patterns, derived from 100 years of simulated data, is similar to the pair of patterns derived from observed data, with an anomalous southwesterly flow being associated with an overall warming of the order of one to two degrees.
- The details, however, do not fit. First, the correlation is only 0.53 compared to 0.64 in the real world. Second, the structure within Central Europe is not reproduced: Maximum temperature anomalies are at the westernmost grid points and minimum values at the easternmost. The local explained variances η are much higher for the GCM output (with a maximum of 96% and a minimum of 50% compared to 58% and 22% in the real world).



First pair of CC patterns derived from the output of a climate model. Left: SLP pattern; right: regional temperature at 6 grid points located in central Europe.

A Downscaling Example: Flowering of Snowdrops

- Maak, K. and H. von Storch, 1997: Statistical downscaling of monthly mean air temperature to the beginning of the flowering of Galanthus nivalis L. in Northern Germany. - Intern. J. Biometeor. (in press)
- \vec{X} is the date of first flowering of snowdrops at many locations in Schleswig Holstein.
- \vec{Y} is the European temperature in January, February and March.
- A robust and plausible CCA pair is found.
- A regression model is built, specifying the flowering date as a linear function of the temperature pattern.
- A regression model was used to reconstruct past flowering dates, and to interpret climate change scenario.

Conclusion

- **Pattern-related techniques** are powerful for reducing the degrees of freedom in high-dimensional problems. Various different types of patterns are available.
- **EOFs or Principal Component Analysis** is a powerful technique to decompose the variance of vector time series, so that a minimum of patterns represents a maximum of variance.
- **CCA** is a powerful technique for the identification of linear links between two vector time series.
A related technique is “redundancy analysis”.
- CCA may be used for the design of regression models, which may be used for, for instance, downscaling purposes or for the reconstruction of unobserved states.

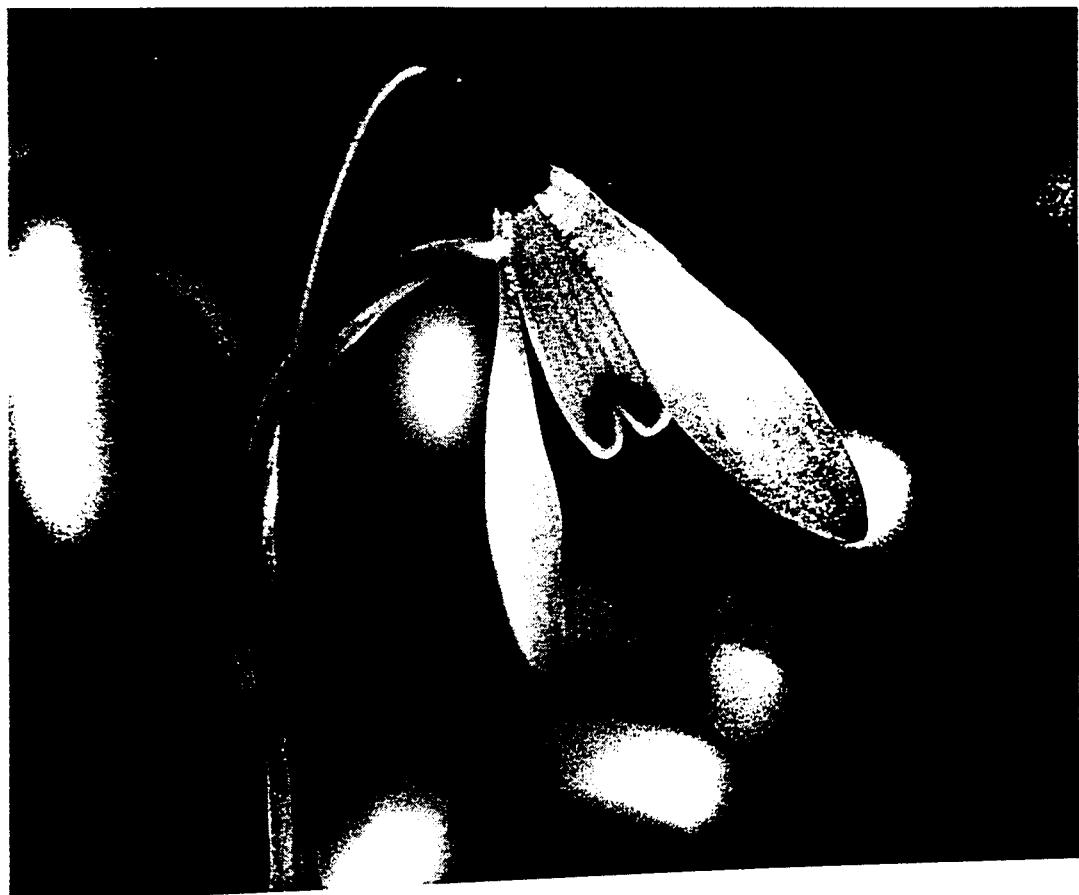


Figure 1: *The flower of the snowdrop.*

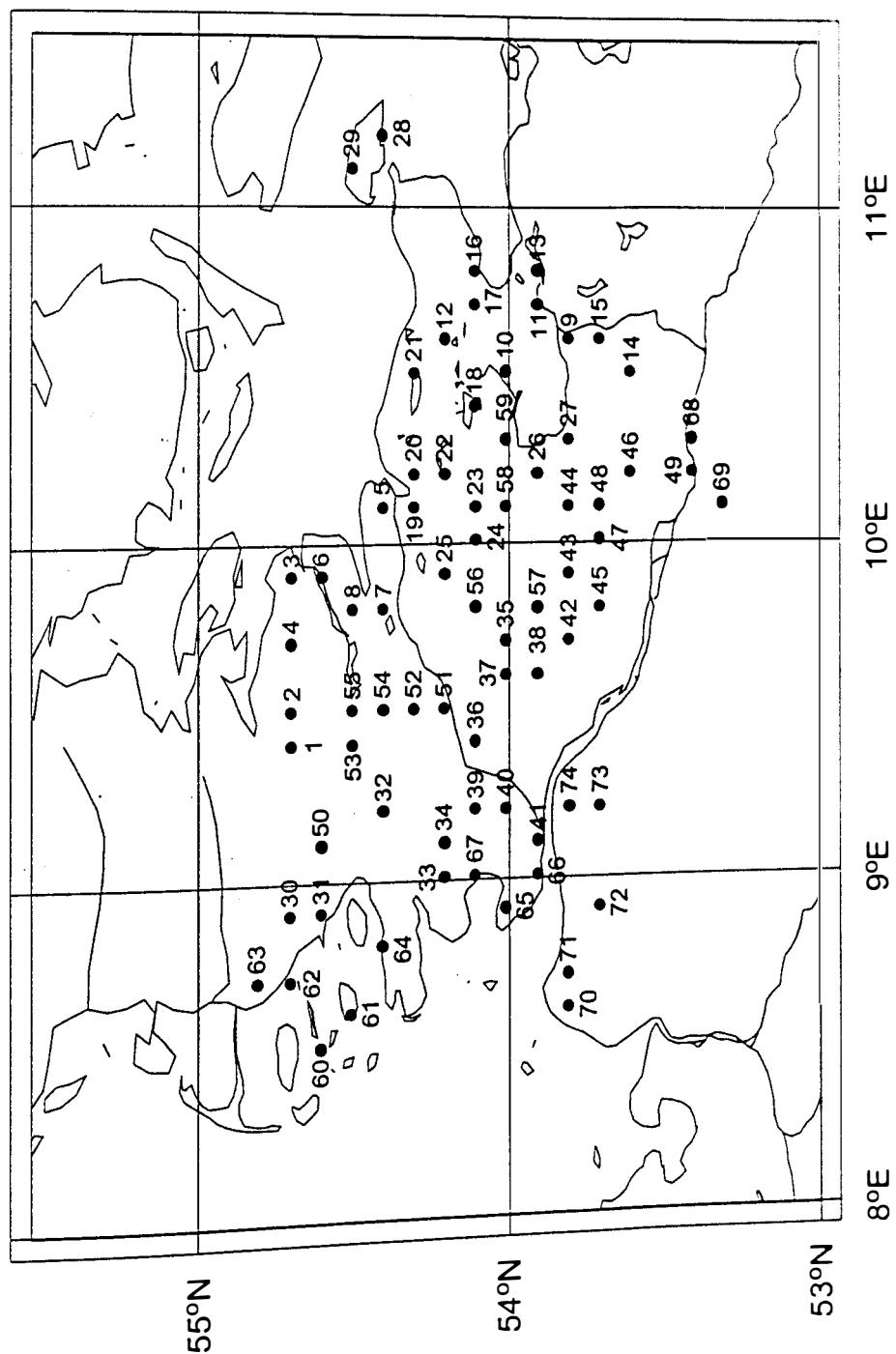


Figure 2: *The distribution of the 74 stations in Schleswig-Holstein.*

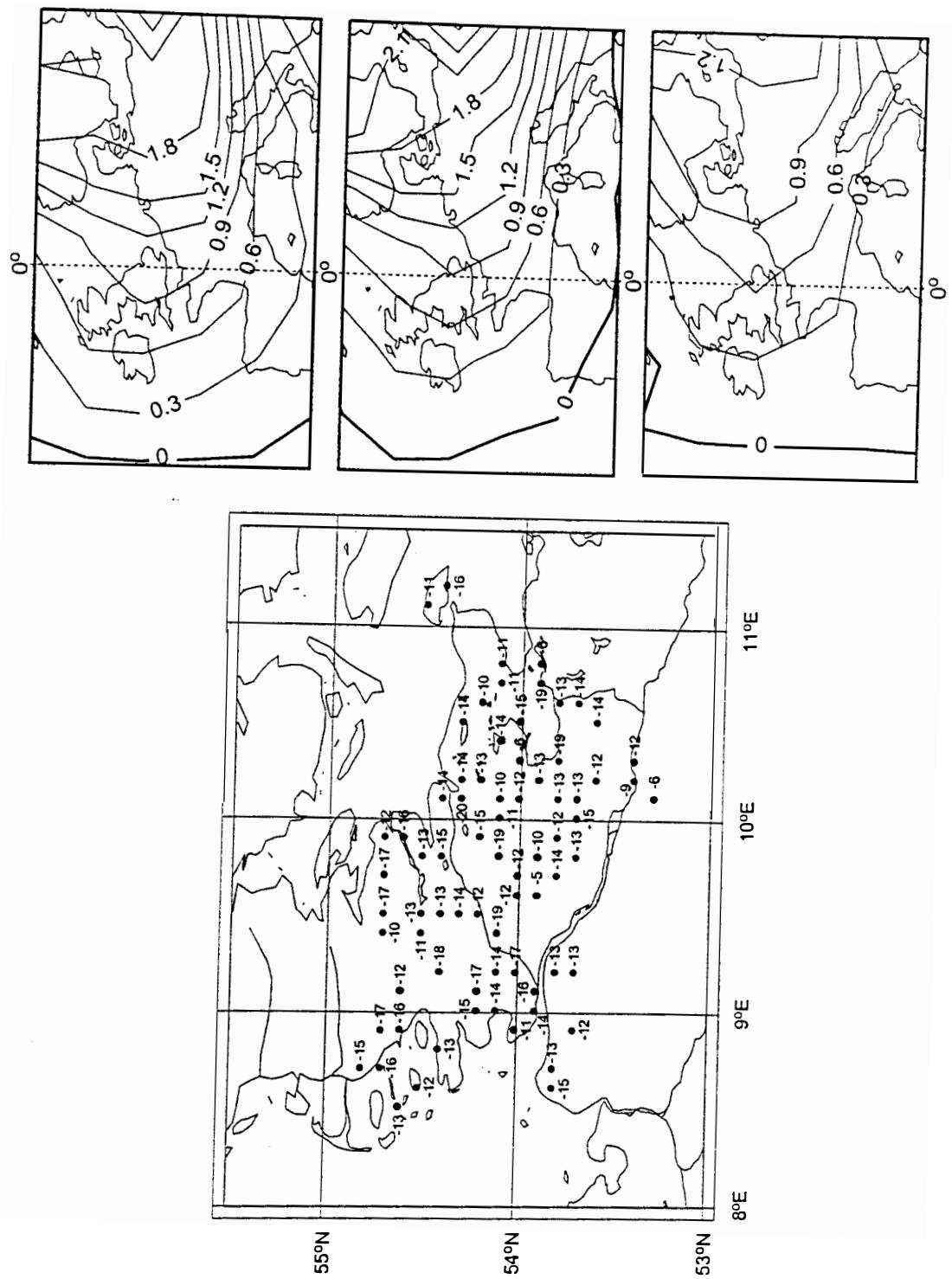


Figure 3: First pair of CCA-pattern of the monthly mean air temperature for January, February and March (from top to bottom) and of the flowering date. The patterns depict anomalies in $^{\circ}\text{C}$ and days. The analysis was done with data in the period 1971 - 1990. For this time interval, the coefficient time series are correlated with 0.96.

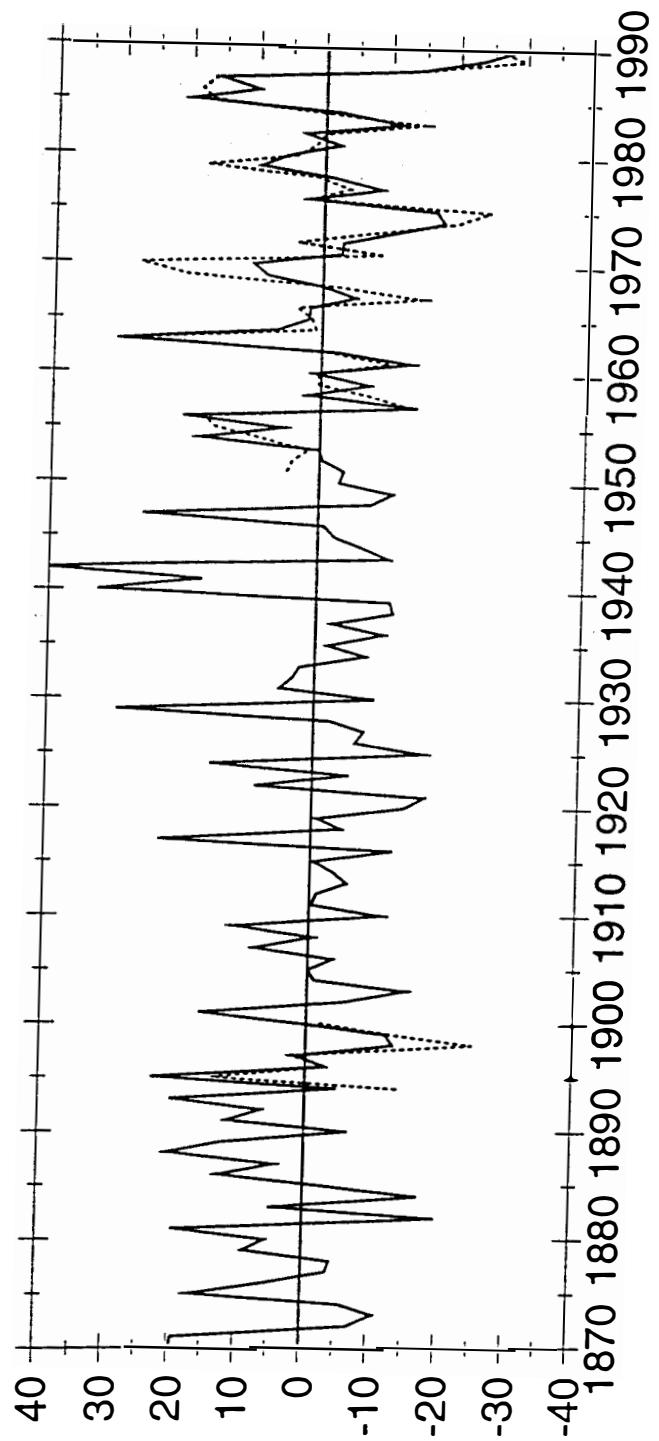


Figure 4: Air temperature observations within 1870 – 1970 are downscaled to derive flowering date anomalies (solid line). These are compared with flowering date observations (dashed lines) from the DWD for the period 1951 – 1970 and from Knuth for the period 1894 – 1900. Furthermore the fitting period (1971 – 1990) is added. The time series depict anomalies in days.

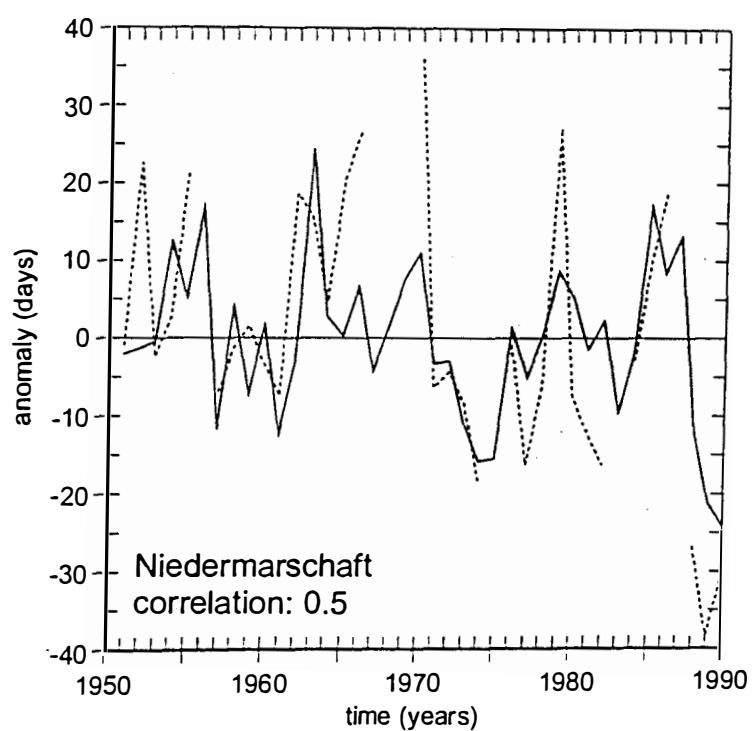
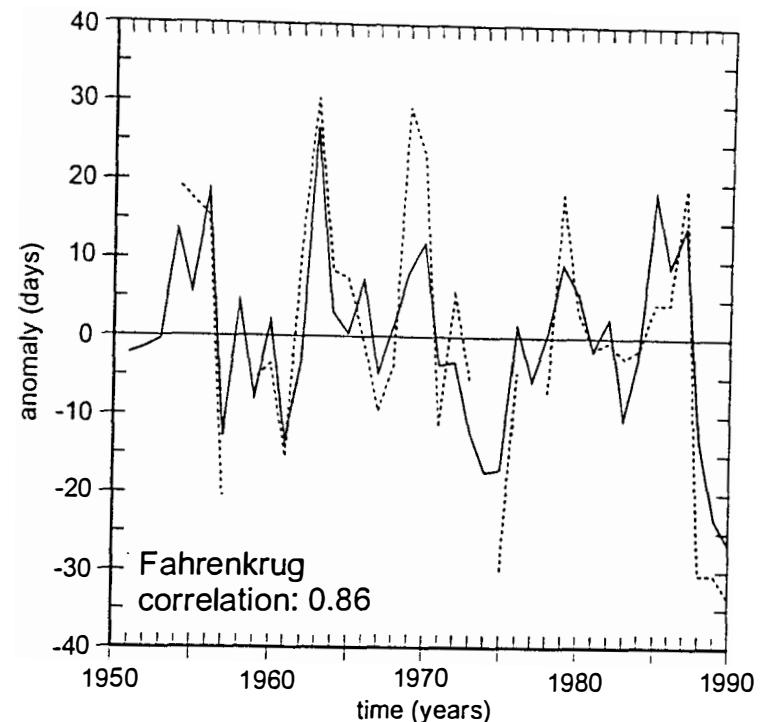


Figure 5: Estimated (solid line) and observed (dashed line) flowering date anomalies for two selected stations, showing a good and a bad match.

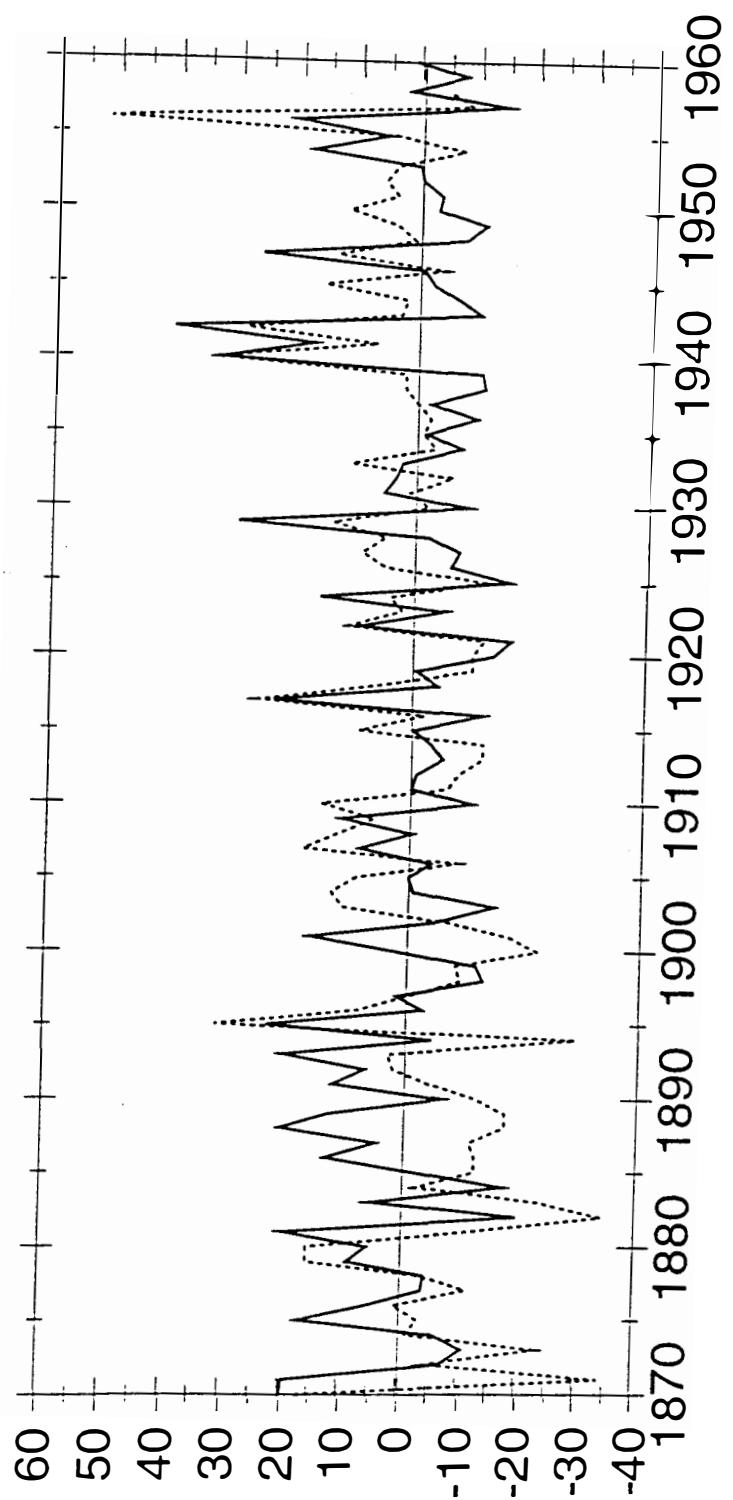


Figure 6: *Estimated historical flowering date variations (solid line) compared with the Marsham flowering date variations (dashed line) from Eastern England. The time series depict anomalies in days.*

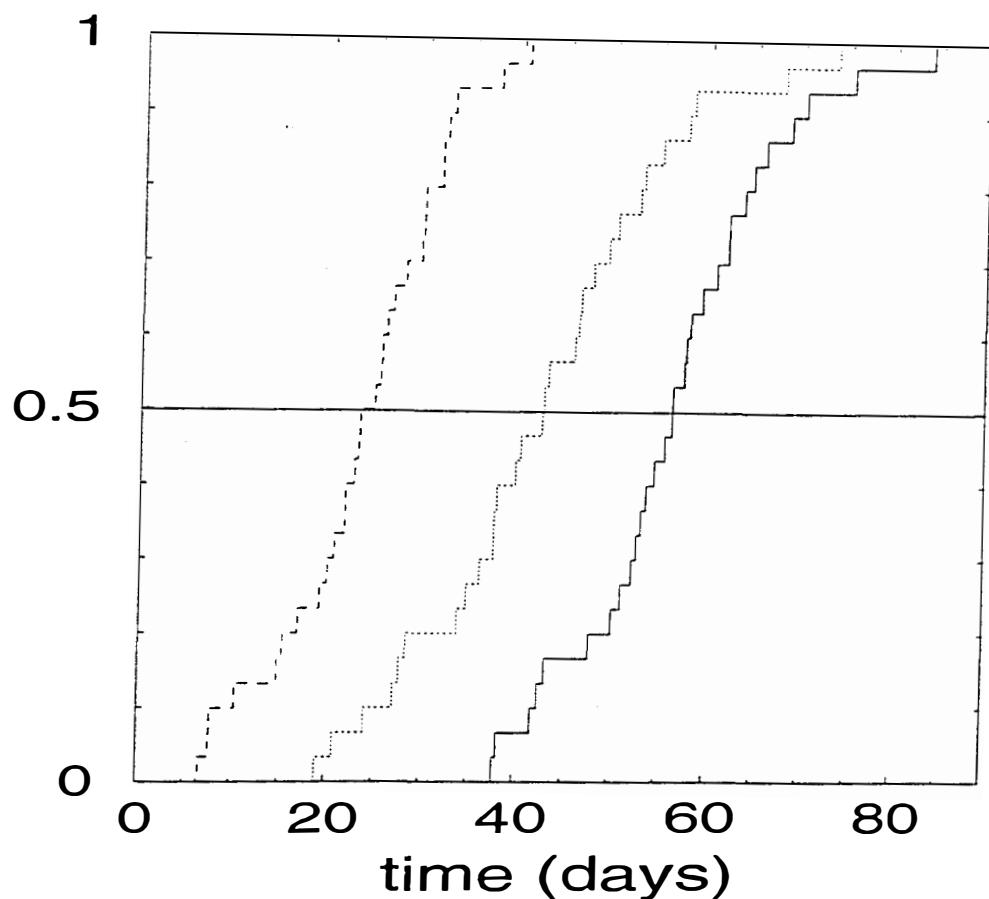


Figure 7: Cumulative frequencies for the estimated flowering date from the air temperature of Jones and Briffa (solid line), the $2 \cdot \text{CO}_2$ time-slice experiment (dotted line) and the $3 \cdot \text{CO}_2$ time-slice experiment (dashed line).