Conditional Statistical Models: A Discourse about the Local Scale in Climate Simulations

Hans von Storch
Institute of Hydrophysics, GKSS Research Centre, Germany

Abstract. The local scale of climate plays two different roles; it is the scale at which people experience climate, so that it is the dominant scale of applied climate research, ranging from climate impact to forecasting weather in the atmosphere and the ocean. On the other hand, the local scale is not important in its details for the formation of the global climate. For the understanding, and simulation, of the global climate, the small scales matter only in a statistical sense so that their influence may be described by means of parameterizations. In the present essay, we demonstrate that both processes, "downscaling" (the derivation of local information in climate change and climate variability simulations and in weather forecasts) and "parameterization" (the description of the net effect of small scales on the larger scales) may formally be understood as the building of empirical models whose parameters are conditioned upon larger-scale features of the state of the atmosphere or ocean. It is suggested to acknowledge the presence of unknown processes by building downscaling and parameterization procedures with a randomized design, conditioned upon the known resolved scales.

1. Introduction

1.1. The Character of the Local

The present essay is about the "local" in climate, which we may vaguely define for the time being as the scale on which we, as people, experience climate, but also as that scale which is far too small to ever be globally captured in detail in climate models. Thus, "local" may mean scales of meters to several kilometers.

By this definition, the local cannot be described by climate models, which would be disastrous if the *global climate were the sum of all local climates.* Luckily, the global climate is *not* the sum of local climates, but may be understood as being formed in a cascade of decreasing spatial scales: To first order approximation, the global climate is the response of the fluid ocean and fluid atmosphere to the differential interception of solar radiation, which is modified by the presence of planetary scale differences in surface properties such as coverage by the ocean, presence of mountains and the like. This global picture is then further modified by features that result from interaction between the planetary scale atmospheric and oceanic flows and the local features such as marginal seas, secondary mountain ranges, differences in land use, and so forth.

In general, the statistics of climate variability at a certain scale are conditioned by the state of the climate system at larger scales, whereas variations on smaller scales are in most cases insignificant in their details but their influence may be captured by some overall statistics. When building realistic climate models, the effect of the sub-scale processes, such as convection in the ocean or momentum flux through the interface of atmosphere and ocean, is *parameterized.* In Section 3 we will discuss the paradigm of parameterizations with the help of an example and point out a routinely made assumption, namely that only those influences that can be conditioned upon features on resolved spatial scales matter for the development of the system. We will argue that this assumption is inadequate and that it can be easily overcome by a randomized design of parameterizations.

The presence of the cascade of decreasing spatial scales, as well as the summary description of the subgrid scale processes, implies that the skill of climate models also decreases with spatial scales and that such models have no skill left at the local scale. This loss of skill is inconsequential for the modelling of the planetary scales and for our understanding of the climate engine. It is, however, consequential for users of climate information.

Our statement that the global climate is not the sum of all local climates is valid only as long as we consider "climate" as a scientific object; if we consider "climate" as the everyday experience of people, which conditions many aspects of life to some extent, then the definition of the global climate as the sum of all local climates is meaningful. The socially defined item "climate" is significantly different from the item "climate" as understood by natural sciences (cf. von Storch and Stehr, 1997).

Thus we are in a strange situation. Our climate models are capable of simulating the overall performance of the climate engine. As such, climatologists are able to answer the fundamental questions of society about climate. However, for applications, which are almost always on the local scales, the climate models have no or few answers. Special efforts are needed to deduce information at the small scale from knowledge about large scales. We discuss the basic "downscaling" approach in Section 2 and illustrate it with an example.

When comparing the two very different tasks, namely the parameterization of subgrid scale processes and the downscaling of large-scale features for estimating local variability, it turns out that the tasks are conceptually similar. In both cases, the basic approach is to describe the local variability as a random variable with a few parameters that are directly linked to the large (resolved) scale.

1.2. Conditional Statistical Models

If a random variable $\vec{\mathbf{X}}$ is *conditioned* upon another random variable $\vec{\mathbf{G}}$ then the probability density function $F_{\vec{X}}(\vec{x})$ of $\vec{\mathbf{X}}$ may by partitioned such that

$$f_{\vec{X}}(\vec{\mathbf{x}}) = \int f_{\vec{X}|\vec{G}=\vec{g}}(\vec{\mathbf{x}}) f_G(\vec{\mathbf{g}}) d\vec{\mathbf{g}} \tag{1}$$

where $f_{\vec{X}|\vec{G}=\vec{g}}(\vec{x})$ is the *conditional* probability function of $\vec{\mathbf{X}}$ provided that the random variable $\vec{\mathbf{G}}$ takes the value $\vec{\mathbf{g}}$, and $f_{\vec{G}}$ is the probability density function of $\vec{\mathbf{G}}$ (cf. Katz and Parlange, 1996).

The expectation and the variance of $\vec{\mathbf{X}}$ may be decomposed:

$$\mathrm{E}_X\left(\vec{\mathbf{X}}\right) = \mathrm{E}_G\left(\mathrm{E}_X\left(\vec{\mathbf{X}}|\vec{\mathbf{G}}\right)\right) \tag{2}$$

$$\begin{aligned}\mathrm{VAR}_X\left(\vec{\mathbf{X}}\right) &= \mathrm{E}_G\left(\mathrm{VAR}_X\left(\vec{\mathbf{X}}|\vec{\mathbf{G}}\right)\right) \\ &\quad + \mathrm{VAR}_G\left(\mathrm{E}_X\left(\vec{\mathbf{X}}|\vec{\mathbf{G}}\right)\right)\end{aligned} \tag{3}$$

where the subscript indicates the random variable upon which random "expectation" and "variance" are operating.

Thus, the overall expected value of $\vec{\mathbf{X}}$ is a weighted mean of the conditional expectations; the overall variance is seen to be attributable to two different sources, namely the mean uncertainty of the various conditional distributions and the variability of the different conditional means.

In the following, we will argue that most downscaling and parameterization prescriptions underestimate the overall variance because of a disregard of the first term in (3).

For further demonstration of this effect, let us consider the regression case. To do so, we assume that the univariate variable $\mathbf{X}$ is normally distributed with mean μ and variance σ_x^2: $\mathbf{X} \sim \mathcal{N}(\mu, \sigma_x^2)$. Let us further assume that the mean state μ depends on a large-scale state $\mathbf{G}_t$ through a linear relationship

$$\mu = \mu_0 + \beta \mathbf{G} \tag{4}$$

and that the variability around μ is independent of $\mathbf{G}$. Then

$$\mathbf{X}_t = \mu_0 + \beta \mathbf{G}_t + \mathbf{N}_t \tag{5}$$

with a normally distributed variable $\mathbf{N} \sim \mathcal{N}(0, \sigma_n^2)$. If the "driving" process $\mathbf{G} \sim \mathcal{N}(0, \sigma_g^2)$, then

$$\begin{aligned}\mathrm{E}(\mathbf{X}) &= \mu_0 \\ \mathrm{E}(\mathbf{X}|\mathbf{G}_t) &= \mu_0 + \beta \mathbf{G}_t \\ \mathrm{VAR}(\mathbf{X}) &= \mathrm{E}\left((\mathbf{X} - \mu_0)^2\right) \\ &= \mathrm{E}\left((\beta \mathbf{G}_t + \mathbf{N}_t)^2\right) \\ &= \beta^2 \sigma_g^2 + \sigma_n^2\end{aligned} \tag{6}$$

This decomposition is a special version of equation (3) and attributes part of the $\mathbf{X}$-variance to the internal variability (σ_n^2) unrelated to the driving process and the remaining variance to the variability of the driving process itself (σ_g^2).

2. The Local as Object of Interest

In this section we will discuss the problem of how to infer local details from information about the large-scale state. This problem has long been known in the field of weather forecasting (Section 2.1). After the advent of climate models, which simulate in a Monte Carlo manner the statistics of climate, the same problem has arisen in climate research (Section 2.2). We will distinguish between the task of determining a "best guess", which has supposedly the smallest distance to the unknown "true" state, and the task of generating the right variability within a simulated ensemble.

2.1. The Case of Forecasting

The need to discriminate between the dynamics of the large scales and the effect of these large-scale features on the local scale was already acknowledged by weather forecasters in the days before the introduction of numerical weather forecasting, as was spelled out explicitly by Victor Starr in his 1942 monograph about weather forecasting:

"The general problem of forecasting weather conditions may be subdivided ... into two parts. In the first place, it is necessary to predict the state of motion of the atmosphere ... and, secondly, it is necessary to interpret this expected state of motion in terms of the actual weather which it will produce at various locations. The first of these problems is essentially of a dynamics nature ... The second problem involves a large number of details because, under exactly similar conditions of mo-

tion, different weather types may occur, depending upon the temperature of the air involved ... and a host of local influences."

This statement holds also today, after the advent of sophisticated detailed numerical weather prediction models which operate on spatial scales of 100 km and less. The numerical results are post-processed by dynamical or statistical models that relate upper air data to local variables, such as hours of sunshine on small islands off the coast or in valleys which cannot be resolved adequately by the dynamical models. These technique go under the names of "Model Output Statistics" and "Perfect Prog" (e.g., Wilks, 1995). In the former case, statistical models are built which relate the forecasts to the observed values, whereas in the "Perfect Prog" approach such models are built from upper-air or large-scale observations and local variables.

2.2. The Case of Downscaling

What we have said so far means, in the climate (change) context, that the local state of the ocean, atmosphere, or other components of the climate system is given by a random variable

$$\vec{\mathbf{X}}_t \sim \mathcal{P}(\vec{\alpha}|\vec{\mathbf{X}}_{t-1}) \tag{7}$$

with a vector of parameters $\vec{\alpha} = (\alpha_1 \ldots \alpha_K)$. The probability distribution $\mathcal{P}$ has to be chosen from a suitable family of distribution. In many cases $\mathcal{P}$ will be Gaussian so that $\alpha_1 = \vec{\mu}$ and $\alpha_2 = \mathbf{\Sigma}$, with $\vec{\mu}$ representing the mean vector and $\mathbf{\Sigma}$ the variance-covariance matrix. In other cases, $\mathcal{P}$ may be of considerably more complex form, for instance in the case of daily amounts of rainfall (cf. Katz and Parlange, 1996). In the following we will disregard the dependence on the previous local state $\vec{\mathbf{X}}_{t-1}$ for simplicity.

In the framework of downscaling, the parameters are conditioned upon a large scale feature G_t that is believed to be well simulated by a climate model (von Storch et al., 1993)

$$\vec{\alpha} = \mathcal{F}(G_t) \tag{8}$$

so that equation (7) is replaced by

$$\vec{\mathbf{X}}_t \sim \mathcal{P}(\mathcal{F}(G_t)) \tag{9}$$

After the specification of $\mathcal{F}$ random sequences of $\vec{\mathbf{X}}_t$ may be generated by drawing random samples from $\mathcal{P}(\mathcal{F}(G_t))$, with G_t itself drawn from a statistical model (i.e., from a stochastic process $\vec{\mathbf{G}}_t$) or from a global (large-scale) dynamical atmosphere-ocean model.

Thus, we may understand the "local climate" $\vec{\mathbf{X}}$ as being conditioned by the external variable $\vec{\mathbf{G}}$. According to (3), the local climate variability is caused by the local uncertainty unrelated to the large-scale dynamics and by the large-scale variability.

2.3. Best Guess – Reconstructions

The externally induced part of the variability, which is in general

$$\mathrm{VAR}_G\Big(\mathrm{E}_X\Big(\vec{\mathbf{X}}|\vec{\mathbf{G}}\Big)\Big)$$

or, in the linear univariate case (5), $\beta^2\sigma_g^2$, may be recovered by calculating for each external state $\vec{\mathbf{G}}_t$ the conditional expected, or ensemble mean state of $\vec{\mathbf{X}}_t$. This part of the variability may be considered "deterministic" or "predictable" to the extent that $\mathbf{G}_t$ is predictable. The remaining part *cannot* be specified (at least as long as the model $\mathcal{F}$ is chosen correctly) from large-scale information..

A standard statistical approach to this end is regression, which is based on the model (5) and leads to the "best guess"

$$\hat{\mathbf{X}} = \mu_0 + \beta\mathbf{G} \tag{10}$$

In the general terminology of Section 1.2, the statistical model (9) is replaced by the deterministic conditional specification

$$\hat{\mathbf{X}} = \mathrm{E}(\mathcal{P}(\mathcal{F}(\mathbf{G}_t))) \tag{11}$$

When attempting to determine a specific state, be it in the past or in the future, then the estimator (11), which tacitly sets the contribution of the intrinsic uncertainty to zero, is the "best guess" with the minimum expected mean square error. Therefore, this type of downscaling is reasonable for the reconstructing of past variations and for forecasting specific states. Of course, the model (11) also makes sense when the intrinsic uncertainty is negligible, which is sometimes the case when we are dealing with time averages.

An example of such a reconstruction is the first flowering date of the *Galanthus nivalis* (snow drop) in Schleswig Holstein, the northernmost state of Germany (Maak and von Storch, 1997). Thus, the local variable $\mathbf{X}_t$ is the Julian date of the first sight of a flowering *Galanthus nivalis*; as a large-scale parameter, $\vec{\mathbf{G}}_t$, the European scale JFM mean temperature was chosen. The index t counts years. With the help of a canonical correlation analysis (e.g., von Storch, 1995b), a regression model of the type (5) was fitted to the data, covering the years from 1871 to 1990.

Figure 1 shows the reported flowering date anomalies, i.e., deviations from the time mean of the first flowering date, for the years 1896-1900 and 1951-1990 as well as the estimated flowering dates, derived via downscaling from the European JFM temperature field. The local reports 1896-1900 and 1951-70 represent independent information. Obviously, the regression model is skillful in reproducing the observed dates so that the best guess for the years without a local report may be considered useful information about interdecadal variability and the presence, or absence, of trends.

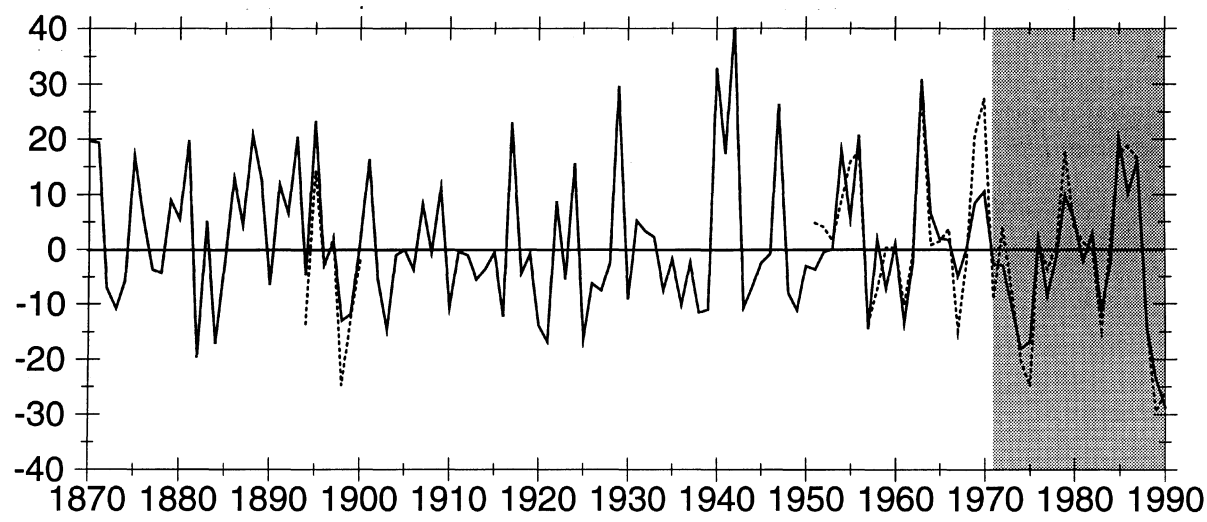

Figure 1. Local reports (dashed) and downscaled anomalies (solid) of the first flowering dates in Schleswig-Holstein. Units: Days.

Similarly good results, which could be verified by independent data from the early part of this century, have been reported by Kaas et al. (1996) for wind force in Greenland and by von Storch et al. (1993) for precipitation on the Iberian peninsula.

2.4. Random Realizations – Simulations

When dealing with the best guess (11), the disregard of the intrinsic local variability has the sometimes disadvantegous side effect that the *time variance* is undestimated:

$$\mathrm{VAR}\left(\hat{\mathbf{X}}\right) < \mathrm{VAR}(\mathbf{X}) \qquad (12)$$

The missing variance is intrinsically unpredictable as long as climate is concerned. It is not a nuisance or due to measurement or modeling errors but an essential part of the statistics of local climate. It is real and significant for ecosystems and people's activity on the local scale. It cannot be accounted for by an inflation of the "deterministic part" as suggested by Karl et al. (1990) or Bürger (1996) but must be kept as independent random component.

Thus, the missing variance must be accepted as it is, namely as variance due to unknown processes, and we replace the downscaling *interpretor* $\hat{\mathbf{X}}$, given by definition (11), with the *randomized* downscaling interpretor, $\mathbf{X}^*$

$$\mathbf{X}^* = \hat{\mathbf{X}} + \mathbf{N} \qquad (13)$$

where $\mathbf{N}$ is assumed to be independent of $\hat{\mathbf{X}}$ so that

$$\mathrm{VAR}(\mathbf{N}) = \mathrm{VAR}(\mathbf{X}) - \mathrm{VAR}\left(\hat{\mathbf{X}}\right) \qquad (14)$$

for having the right variance. In any concrete situation, with a given "deterministic" $\hat{\mathbf{X}}$, nothing more is known about $\mathbf{N}$, only its variance (14) and its independence. Thus in the situation that time series are wanted that reproduce the original variance, then the randomized design (13) is better than the deterministic approach (11).

We return to the case of *Galanthus nivalis* and its first flowering in Schleswig-Holstein. Admittedly, the event "flowering of *Galanthus nivalis*" is not of great public or scientific concern when we address climate change, but expressing the climate change message in terms of this phenological event typical of the experiences of everyday life may be more ilustrative than expressing it in terms of winter mean temperatures in degrees centigrade.

The standard deviation of the in situ dates is 16.5 days ($\mathbf{X}$) and that of the downscaled dates is 14.9 days ($\hat{\mathbf{X}}$). For arriving at the same variance, the additive white noise process must have a standard deviation of 7.1 days.

We have applied the "deterministic" downscaling (11) and the randomized downscaling interpretor (14) to the large-scale temperature fields simulated in a transient climate change experiment, which was initialized with CO_2 concentration as in 1935, run with CO_2 concentrations as observed until 1990, and then continued with a "business-as-usual" scenario of a 1% annual increase of CO_2 until 2090 (Cubasch et al., 1995).

The result of the downscaling exercise is shown in Figure 2 with two independent realizations of the randomized interpretation of the large-scale temperature. The three curves share essentially the same behavior, namely an almost stationary behaviour until the end of this century and then a smooth transistion toward earlier and earlier flowering dates; after about 100 years, the mean flowering date is indicated to happen about 30 days earlier than in the 20th century.

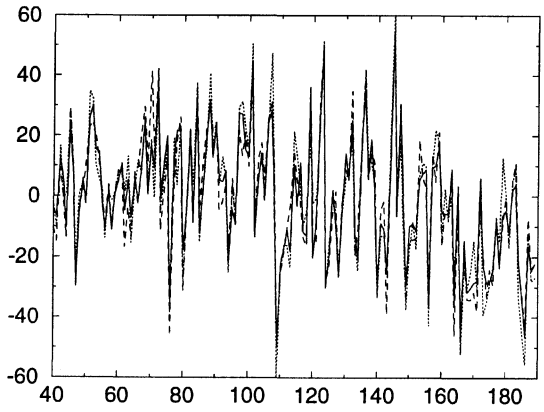

Figure 2. Scenarios of first flowering of *Galanthus nivalis* in Schleswig Holstein derived from a 150-year climate change experiment starting in 1935. The solid line specifies the conditional expectation whereas the dashed and dotted lines represent two randomized scenarios with independent noise terms with a standard deviation of 7 days. Horizontal axis: years (+ 1900), vertical axis: Julian days.

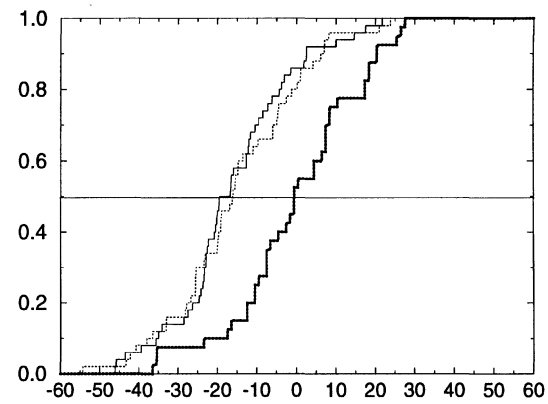

Figure 3. Cumulative distribution function derived from in situ observations (heavy solid line) and estimated doubled carbon dioxide conditions without (light solid line) and with (dotted) superimposed white noise (with standard deviation 7 days).

The difference between the deterministic and randomized estimates is small, which is not surprising because the standard deviation of the noise term was only about half of the $\hat{\mathbf{X}}$-variability in the control climate. That the randomization has, however, an effect is demonstrated by the cumulative distribution functions shown in Figure 3. In this case, 30 different winter temperature distributions taken from a T42 time slice experiment with present day observed CO_2 and with doubled CO_2 concentrations were downscaled to get estimates of *Galanthus nivalis* flowering dates. Thus, we get 30 realizations of mean flowering dates conditioned upon a temperature field. The cumulative distribution functions for the two cases are given as heavy (present day) and light (2 CO_2) solid lines—the distribution is clearly shifted by 20 days in the mean. When, however, for the double CO_2 case, the randomization is introduced (dotted line), the mean value is almost unchanged, but the distribution becomes markedly broader (see the tails of the distribution).

2.5. Weather Generators

This example may be considered a simple case of a "weather generator", i.e., a presciption to generate sequences of numbers which share certain statistics of the real world. Of course, considerably more sophisticated generators may be designed, as for instance the precipitation generator proposed by Katz and Parlange (1996), in which the distributions of both the amount of precipitation and the probability for a day to be wet after the day before was already wet (or dry) are conditioned upon the large-scale atmospheric state.

Another class of weather generators was suggested by Hughes et al. (1993), who considered the vector formed by a few indices characteristic for the air pressure distribution[1] *and* the daily precipitation. With the help of a clustering procedure (CART), they partitioned the full space spanned by the air-pressure indices into a few subsets which were found to be associated with a characteristic rainfall regime. After having determined the subsets, the procedure was applied to air pressure index vector sequences simulated by a climate model. The purpose of this exercise is the generation of realistic, dynamically consistent rainfall sequences. To do so, they estimated for each day in the subset of air pressure a consistent rainfall by choosing at random one day from the large observational data set that fell in the same subset of the air pressure indices.

Yet another approach, the "analogue technique" was suggested by Zorita et al. (1995), who also formed a vector with indices of the large-scale air pressure field. For any climate model air pressure distribution, they determined from the large set of observed weather maps which map would be closest to the examined simulated map. Then, for the climate model day, the local feature—be it rainfall or high water level at a coastal gauge—was specified as being the local feature of the nearest neighbor in the set of observed states.[2] The result of such an exercise is shown in Figure 4: In this case the local variable is the daily mean height of the high tides at the North German port Cuxhaven. The top diagram shows the daily reports for 10 consecutive winters (DJF), and the two diagrams below show the analogue specifications (nearest and 2nd nearest neighbor). The specifications deviate from the reports quite a bit in the details, but the overall statistics, with times of increased and reduced heights, are clearly reproduced.

[1] Actually, EOF coefficients.

[2] The procedure is slightly more involved, but for the time being this simplifying description should be sufficient.

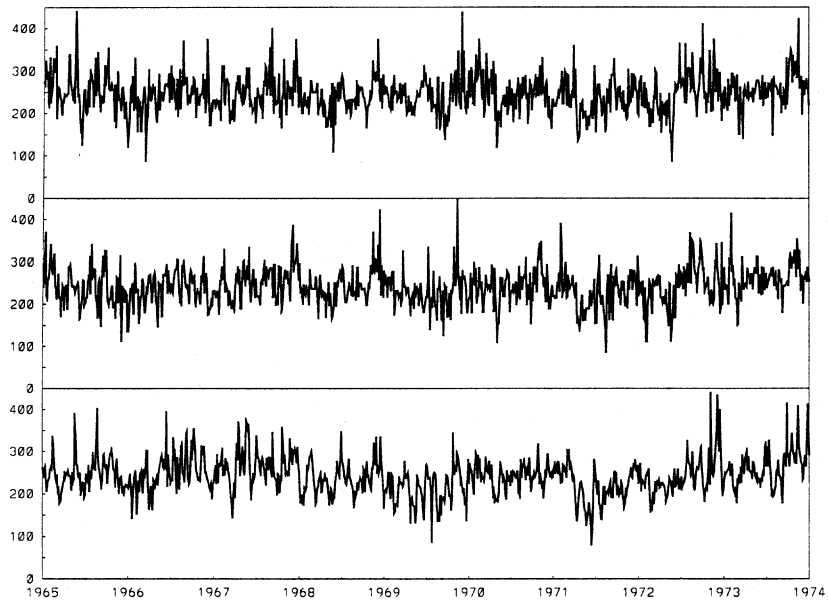

Figure 4. With an "analogue weather generator" created sequences of daily mean high-tides (cm) during 10 winters (DJF; 1 January 1965 until 31 December 1974) in Cuxhaven. Top: reported values; lower two panels: specifications with the nearest and 2nd nearest neighbor.

3. The Local Modifying the Global

So far, we have dicussed the problem of inferring details of climate statistics on scales where climate models have lost their skill. But what is the effect of these unresolved scales on the general performance of the climate engine? Let us write a climate variable ϕ as a sum of the large-scale resolved component $\bar{\phi}$ and an unresolved part ϕ'

$$\phi = \bar{\phi} + \phi' \tag{15}$$

Then, our basic differential equations

$$\frac{\partial \phi}{\partial t} = \mathcal{R}(\phi) \tag{16}$$

is replaced by

$$\frac{\partial \bar{\phi}}{\partial t} = \mathcal{R}_{\Delta x}(\phi) \tag{17}$$

with a modified operator $\mathcal{R}_{\Delta x}$ resulting from the full operator $\mathcal{R}$ after introducing a truncated spatial resolution Δx (this could be the global or regional average). In general, this operator may be written as

$$\mathcal{R}_{\Delta x}(\phi) = \mathcal{R}(\bar{\phi}) + \mathcal{R}'(\phi') \tag{18}$$

with an operator $\mathcal{R}'$ describing the net effect of the subgrid scale variations represented by ϕ'. With this setup, the system (17) is no longer closed and can therefore no longer be integrated. For overcoming this problem, conventional approaches assume that the "nuisance" term $\mathcal{R}'(\phi')$ is either irrelevant, i.e,.

$$\mathcal{R}'(\phi') = 0 \tag{19}$$

or may be *parameterized* by

$$\mathcal{R}'(\phi') = \mathcal{Q}(\bar{\phi}) \tag{20}$$

with some empirically determined or dynamically motivated function $\mathcal{Q}$.

While both specifications (19,20) return an integrable equation (17), they both have to assume that the local scale acts as a deterministic slave of the resolved scales. However, as we have seen before, in reality there is variability at local scales *unrelated to the resolved scales.* Thus, equation (17) should take into account that $\mathcal{R}'(\phi')$ can not completely be specified as a function of $\bar{\phi}$, but that formulation (20) should be replaced by

$$\mathcal{R}'(\phi') \sim \mathcal{S}(\vec{\alpha}) \tag{21}$$

with a random process $\mathcal{S}$ with parameters $\vec{\alpha}$ which are conditioned upon the resolved state $\bar{\phi}$:

$$\mathcal{R}'(\phi') \sim \mathcal{S}(\mathcal{F}(\bar{\phi})) \tag{22}$$

When the mean value μ is the only parameter in the vector $\vec{\alpha}$ which depends on $\bar{\phi}$, then the distribution $\mathcal{S}$ may be written as

$$\mathcal{S}(\mathcal{F}(\bar{\phi})) = \mu(\bar{\phi}) + \mathcal{S}' \tag{23}$$

with a conditional mean value and random components with zero mean value ($\mathrm{E}(\mathcal{S}') = 0$) and uncertainty unrelated to the resolved scales. Specification (20) equals specification (23) if $\mathcal{S}' = 0$ and $\mu(\bar{\phi}) = \mathcal{Q}(\bar{\phi})$.

To demonstrate the difference between the two specifications (20) and (23), we discuss Figure 5 display-

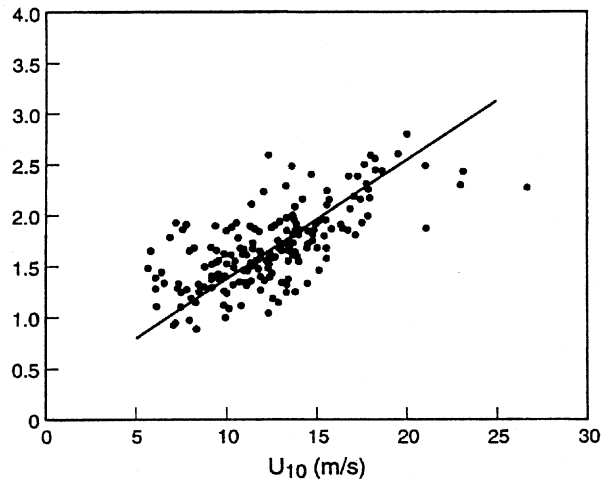

Figure 5. Scatter of various simultaneous measurements of the drag coefficient c_D and of the wind speed at 10 m height. The straight line is a regression line for the scatter. After De Cosmo et al. (1996). [1 mm]

ing various "measurements" of the drag coefficients c_D of the sea surface sorted according to the value of the "10 m wind" $|\vec{u}_{10}|$ during neutral conditions (from De Cosmo et al., 1996). In this case we consider the wind measured at a height of 10 m as the "resolved scale" parameter $\bar{\phi}$, which is representative for a certain spatial and temporal scale and readily observable. The transfer of momentum $\vec{\tau} = \mathcal{R}(\phi')$ through the interface of ocean and atmosphere, however, depends on the variance of short-term and smallest-scale variations of the wind. The latter quantity can be determined only in expensive obervational campaigns, but it has long been known that it can be approximated by the "bulk formula"

$$\vec{\tau} \approx c_D \rho |\vec{u}_{10}| \vec{u}_{10} \tag{24}$$

where c_D is considered to depend on the thermal stability and the wind speed. Formula (24) represents a classical case of a parameterization, namely the frictional effect of small-scale short-term fluctuations of wind on the atmospheric flow. Figure 5 displays a scatter of points, each representing one observation, and a summarizing regression line. Thus, to completely specify the parameterization (24), disregarding the dependency on the thermal stratification for the time being, the drag coefficient is specified as a linear function of $|\vec{u}_{10}|$, namely

$$\widehat{c_D} = a + b \cdot |\vec{u}_{10}| \tag{25}$$

with some, in the present context, irrelevant numbers a and b. This type of specification has been used in countless simulations with numerical ocean circulation models and ocean surface wave models forced with observed wind fields.

The question is what to do about the scatter around the regression line in Figure 5. The application of the bulk formula (24) with the regressed $\widehat{c_D}$ implies that the scatter is considered inconsequential or artificially reflects observational errors. The alternative interpretation is, however, that unknown processes (such as the sea state, gustiness of the wind, secondary flows) influence the value of c_D so that it exhibits unaccounted variations almost symmetrically around the regression line with a standard deviation of about $\sigma \approx 0.5$ units. Therefore, the randomized version of the bulk formula (24) would use

$$c_D = \widehat{c_D} + \mathbf{N} \tag{26}$$

with

$$\mathbf{N} \sim \mathcal{N}(0, \sigma) \tag{27}$$

provided that the bulk formula is used with sufficiently large time steps so that the temporal correlation may be disregarded.

3.1. Does It Matter? A Demonstration

The question is, of course, does the use of the randomized parameterization (23) have an effect on the performance of the model in which the parameterization is used? One may argue, that the introduction of noise, represented by $\mathcal{S}'$, is inconsequential since the contributions will just be averaged out. This may indeed be so in many cases, in particular in diffusive systems, but the situation is similar to the case of the "stochastic climate model" (Hasselmann, 1976), where the noise acts constructively in building up red noise variance. We argue that the use of the randomized design (23) will enhance the variability of the considered model on all time scales and will demonstrate this prospect by means of an energy balance model

$$\frac{d\bar{T}}{dt} = c_w(S_t + L_t) \tag{28}$$

where T represents the near surface temperature and $\bar{T}$ the global mean of T. S is the globally averaged short wave radiation intercepted by the surface, which is determined by a range of factors, such as the extent of sea ice coverage, the cloud distribution, and surface properties. L is the outgoing long wave radiation, which depends on the temperature. Parameterizations of the type (20) are

$$S(\bar{T}) = (1 - A)S_0 \tag{29}$$

$$L(\bar{T}) = b\bar{T}^4 \tag{30}$$

with the albedo A, which is the fraction of the total incoming short wave radiation S_0 reflected to space. The specification of the long-wave back radiation features a constant b which is supposed to represent a diversity of factors, such as surface properties and the presence of radiatively active gases. The dependency of the albedo on the temperature may be approximated by

$$A(\bar{T}) = h - g \cdot \tanh\left(r(\bar{T} - T_0)\right) \qquad (31)$$

with appropriately chosen constants h, g, r and T_0. Large global mean temperatures are assumed to be associated with little or no sea ice, thus low albedo, whereas low temperatures are connected with a large sea ice coverage and high albedo.

The equation has three equilibria (Figure 7), two of which are stable and the third is unstable; whatever the initial state, the system moves smoothly with exponentially decreasing speed towards the nearest stable equilibrium (Figure 6) with no internal variability.

The albedo is not only a function of the global mean temperature but is affected by the cloudiness, which

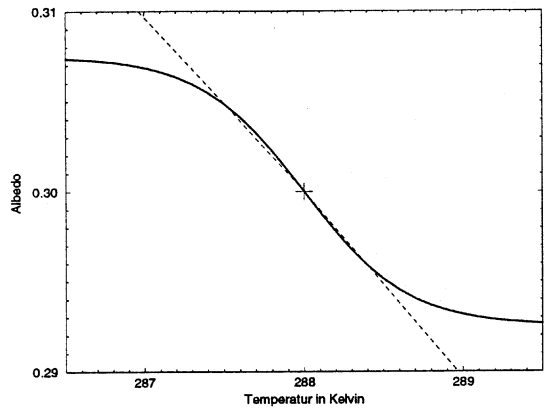

Figure 6. Temperature dependency of the expectation of albedo in the EBM (solid line). The dashed line represents required albedo values for the system to equilibrate at a given temperature.

to some extent is also determined by the global mean temperature but exhibits a marked internal variability related to the regional and local temperatures. If we assume that this internal variability leads to an uncertainty of 3% in (31), then we replace the "deterministic" parameterization (29) by the randomized parameterization

$$S(\bar{T}) \sim (1 - A(\bar{T}) + \mathbf{N})S_0 \qquad (32)$$

with a random variable $\mathbf{N} \sim \mathcal{N}(0,\ 3\%)$. The term $A(\bar{T})$ is the mean albedo, averaged over all cases with global mean temperature $\bar{T}$, given by specification (31). The effect of the randomized albedo parameterization on the global mean temperature $\bar{T}$ is shown in Figure 8.

Note that the resulting model itself has become a stochastic model, since it will deliver a different path in its phase space for each simulation. Differently from the smooth convergence towards one of the stable equilibria in Figure 7, the trajectory wanders irregularly between two regimes, which correspond to the two stable equilibria of the non-randomized system. Overlaid are short-term erratic variations around these equilibira. The existence of the two statistical equilibria is documented clearly from the bimodal histogram of the temperature shown in the lower panel of Figure 8. Thus, the addition of noise transforms the simple dynamically inactive deterministic system into a much richer dynamically active system.

The noise does not act as a nuisance, or a veil blurring the dynamics of the system; instead the noise contributes a significant component of the dynamical system. Therefore, it appears likely that the randomization of subgrid scale parameterizations in numerical dynamical models will have an effect on the simulated space-time statistics; in particular, one may expect the overall level of variability to be enhanced and that more often transitions between different subregimes, if they exist, will occur. Also, the extreme values may become somewhat larger.

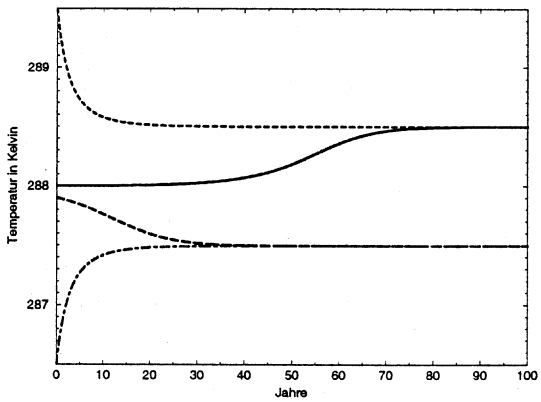

Figure 7. Solutions of the EBM with deterministic parameterization for different initial values. Note that the trajectory launched at 288 K starts out from an unstable equilibrium.

4. Conclusion

The purpose of the present paper is to discuss the role of the "local" in numerical models of the ocean's or atmosphere's dynamics. Conventionally, the local is considered either irrelevant or a complete slave of the resolved scales so that the effect of sub-grid scale dynamics may be approximated by some suitably chosen functions of the resolved scales. As long as such models are considered a tool for describing the large-scale dynamics and the task of parameterizing the subgrid scales is done properly, the statistics of local states are of no interest. However, after numerical models have matured in becoming tools for simulating quasi-realistically the statistics of the physical environment affecting ecosystems and their risk management (e.g., Leemanns and Solomon, 1993; Lubchenco et al., 1993), the implications of model results for the local scales are becoming more and more significant (von Storch, 1995a).

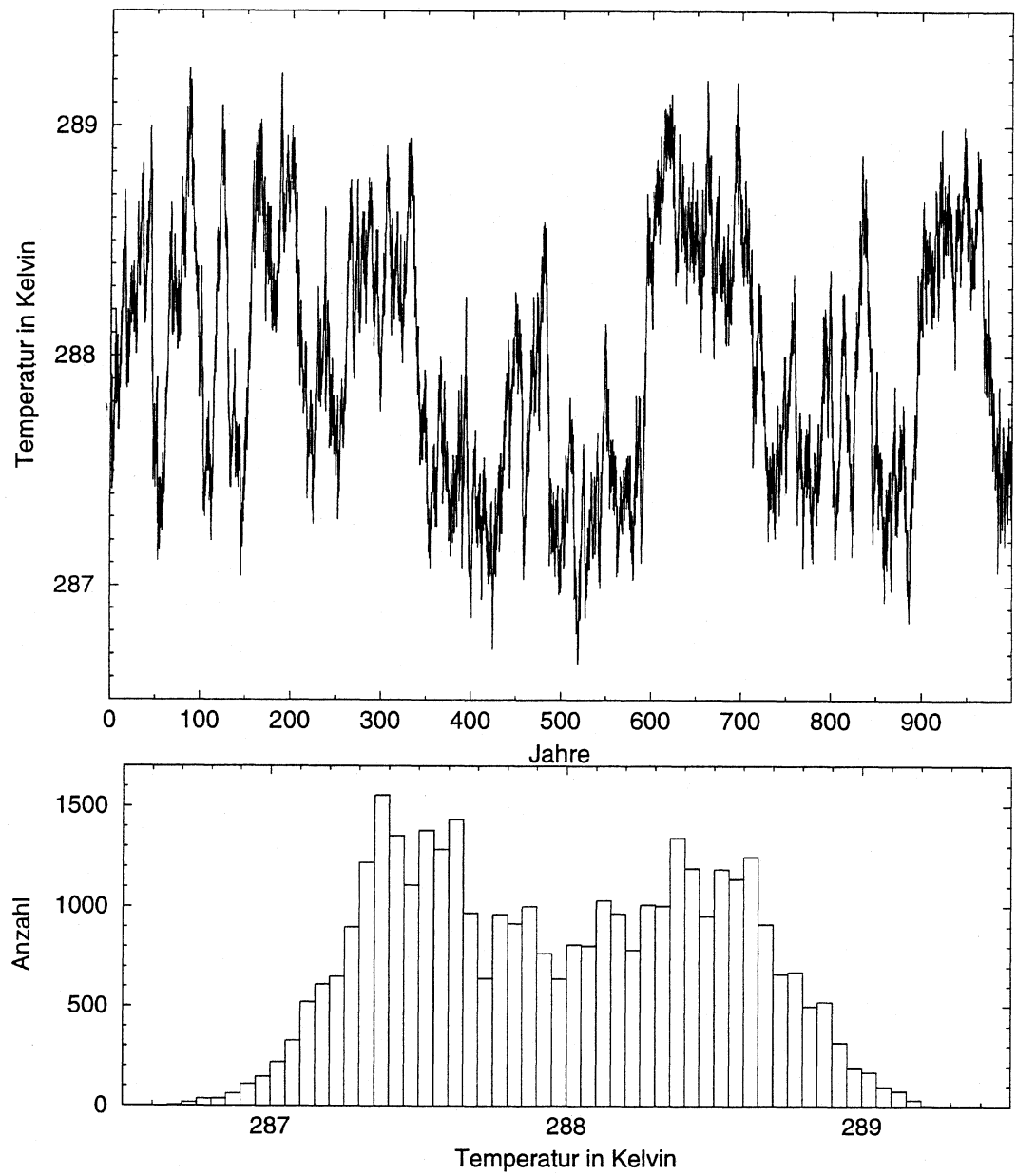

Figure 8. Realization of randomized EBM
Top: time series; Bottom: frequency distribution of the temperature.

In the past five, or so, years, a rich literature in downscaling has emerged (e.g., Wigley et al., 1990; Karl et al., 1990; Hewitson and Crane, 1992, 1996; von Storch et al., 1993; Hughes et al., 1993; Zorita et al., 1995; Frey-Buness et al., 1995; Bürger, 1996; Kaas et al., 1996; Fuentes and Heimann, 1996; Conway et al., 1996; Katz and Parlange, 1996; Cubasch et al., 1996).

On the other hand, the paradigm of specifying parameterizations of sub-grid scale processes as conditional expected values has been adopted in all approaches, without acknowledging this formal dependency. In some cases, researchers have noticed that the sub-grid scale "noise" unrelated to resolved scales has an impact on the resolved scales. Examples are gustiness and its effect on ocean waves (Komen et al., 1995) or on the atmospheric boundary layer. In these cases, the subgrid scale spatial variability was considered by adding a term, which may again depend on the resolved scale (so that the resulting parameterization is again of the deterministic form (20)). The motivation in these cases was that the sub-grid scale variability would act on the resolved scales via a non-linear mechanism, and not via the accumulation of short-term variability. The modification of the parameterization then describes the *conditional mean effect of this nonlinear effect.*

It is suggested to conduct a number of numerical twin experiments with dynamical models using deterministic parameterizations and randomized parameterizations.

It is expected that the randomized parameterizations will give rise to broader frequency distributions with longer tails. It remains to be seen if this prediction will be confirmed or if the diffferences will appear as inconsequential and not worth the additional numerical load.

Acknowledgment. Part of this work was done during an extended visit to Prof. Peter Müller of the Department of Oceanography, University of Hawaii, Honolulu. I thank Peter Müller for his encouragement and financial support for this visit. Stefan Güss and Katrin Maak supplied me with the EBM and the snow drop case and Eduardo Zorita with the Cuxhaven high water simulation. Comments by Kristina Katsaros, Peter Müller, Klaus Hasselmann, Eduardo Zorita, and others were most useful.

References

Bürger, G., Expanded downscaling for generating local weather scenarios, *Clim. Res., 7*, 111-128, 1996.

Cubasch, U., G. Hegerl, A. Hellbach, H. Höck, U. Mikolajewicz, B. D. Santer, and R. Voss, A climate change simulation starting at 1935, *Clim. Dyn., 11*, 71-84, 1995.

Cubasch, U., H. von Storch, J. Waskewitz, and E. Zorita, Estimates of climate change in Southern Europe using differenmt downscaling techniques, *Clim. Res., 7*, 129-149, 1996.

Conway, D., R. L. Wilby, and P. D. Jones, Precipitation and air flow indices over the British Isles, *Clim. Res., 7*, 169-183, 1996.

De Cosmo, J., K. B. Katsaros, S. D. Smith, R. J. Anderson, W. A. Oost, K. Bumke, and H. Chadwick, Air-sea exchange of water vapour and sensible heat: the Humidity EXchange Over the Sea (HEXOS) results, *J. Geophys. Res., 101, C5*, 12,001-12,016, 1996.

Frey-Buness, F., D. Heimann, and R. Sausen, A statistical-dynamcial downscaling procedure for global climate simulations, *Theor. Appl. Climatol., 50*, 117-131, 1995.

Fuentes, U., and D. Heimann, Verification of statistical-dynamical downscaling in the Alpine region, *Clim. Res., 7*, 151-168, 1996.

Hasselmann, K., Stochastic climate models. Part I. Theory, *Tellus, 28*, 473-485, 1976.

Hewitson, B. C., and R. G. Crane, Regional-scale atmospheric controls on local precipitation in tropical Mexico, *Geophys. Res. Letters, 19*, 1835-1838, 1992.

Hewitson, B. C., and R. G. Crane, Climate downscaling: techniques and application, *Clim. Res., 7*, 85-95, 1996.

Hughes, J. P., D. P. Lettenmaier, and P. Guttorp, A stochastic approach for assessing the effect of changes in synoptic circulation patterns on gauge precipitation, *Water Resour. Res., 29*, 3303-3315, 1993.

Kaas, E., T.-S. Li and T. Schmith, Statistical hindcast of wind climatology in the North Atlantic and Northwestern European region, *Clim. Res., 7*, 97-110, 1996.

Karl, T. M., W-C. Wang, M. E. Schlesinger, D. E. Knight, and D. Portman, A method of relating General Circulation Model simulated climate to the observed local climate, Part I: seasonal statistics, *J. Climate, 3*, 1053-1079, 1990.

Katz, R. W., and M. B. Parlange, Mixtures of stochastic processes: applications to statistical downscaling, *Clim. Res., 7*, 185-193, 1996.

Komen, G. J., L. Cavaleri, M. Donelan, K. Hasselmann, S. Hasselmann and P. A. E. M. Janssen, Dynamics and Modelling of Ocean Waves, Cambridge University Press, 532 pp., 1994.

Leemans, R., and A. M. Solomon, Modelling the potential change in yield and distribution of the earth's crops under a warming climate, *Clim. Res., 3*, 79-96, 1993.

Lubchenco, J., S.A. Navarette, B.N. Tissot and J.C. Castilla, Possible ecological responses to global climate change: Nearshore benthic biota of northeastern Pacific coastal ecosystems, in *Earth System Reponses to Global Change*, edited by H. A. Mooney, E. R. Fuentesm and B. I. Kronberg, 147-166, Academic Press, 1993.

Maak, K. and H. von Storch, Statistical downscaling of monthly mean air temperature to the beginning of the flowering of *Galanthus nivalis* L. in Northern Germany, *Intern. J. Biometeor.*, (in press).

Starr, V. P., Basic principles of weather forecasting. Harper & Brothers Publishers New York, London, 299 pp., 1942.

von Storch, H., Inconsistencies at the interface of climate impact studies and global climate research, *Meteor. Z. (NF), 4*, 72-80, 1995a.

von Storch, H., Spatial patterns: EOFs and CCA, in *Analysis of Climate Variability: Applications of Statistical Techniques*, edited by H. von Storch and A. Navarra, p. 227-258, Springer Verlag, 1995b.

von Storch, H., and N. Stehr, The case for the social sciences in climate research, *Ambio*, (in press).

von Storch, H., E. Zorita and U. Cubasch, Downscaling of climate change estimates to regional scales: application to winter rainfall in the Iberian Peninsula, *J. Climate 6*, 1161-1171, 1993.

Wigley, T. M. L., P. D. Jones, K. R. Briffa, and G. Smith, Obtaining subgrid scale information from General Circulation Model output, *J. Geophys. Res., 95*, 1945-1953, 1990.

Wilks, D.S., Statistical Methods in Atmospheric Sciences. Academic Press, San Diego, 1995,

Zorita, E., J. Hughes, D. Lettenmaier, and H. von Storch, Stochastic characterisation of regional circulation patterns for climate model diagnosis and estimation of local precipitation, *J. Climate, 8*, 1023-1042, 1995.