Noise in the Climate System – Ubiquitous, Constitutive and Concealing

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1. Introduction

Climate is largely determined by the fluid flows in the atmosphere and oceans. These flows are governed by the laws of fluid dynamics and thermodynamics. These laws are partial differential equations that represent the conservation of mass, momentum, energy and other quantities.[1] If we could solve these equations, with the right initial and boundary conditions, then we would have the answers to all the pressing questions of the current climate debate. This, however, is not possible. Even if there were a unique set of such equations the only consensus about them is that they are highly and multiply nonlinear. They couple processes across all scales, from the planetary scales of wind and current systems to the micro-scales of molecular diffusion. The resulting flow is turbulent, with everything depending on everything else. It is also impossible to know the exact initial and boundary conditions, such as the exact shape of the continents or the details of human land use.

The climate system is thus a complex system. Another example of a complex system is the Brownian ink particle suspended in a fluid and subjected to the constant bombardment of fluid molecules. Complex systems can be described on different levels. On the micro level one tries to trace the exact time evolution of the system. In case of the Brownian ink particle one tries to follow its exact position. In the case of the climate system one tries to predict the actual weather. On the macro level one is only interested in the typical behavior of the system and considers coarse-grained or averaged quantities. The displacement variance of the Brownian ink particle is such an averaged (and time-independent) quantity. Statistics, like time means and extreme values of temperature and velocity fields in the atmosphere and ocean are such quantities for the climate system. If only a macroscopic description of the system is sought one often replaces the original deterministic system by a stochastic system. The hope is that time averages of the original system are equivalent to ensemble averages of the stochastic system. If this is the case, the system is called ergodic. Probability concepts thus become part of the macroscopic description of complex system. This replacement does

[1] There is, however, a caveat. The differential equations are a limit of discretely formulated principles. It appears questionable that these principles, formulated with continuously distributed variables, such as temperature and velocity, are meaningful for almost infinitely small boxes. Thus, one may argue the differential equations are an approximation of the "real" discrete equations, and not, as is usually perceived by mathematicians, the other way around.

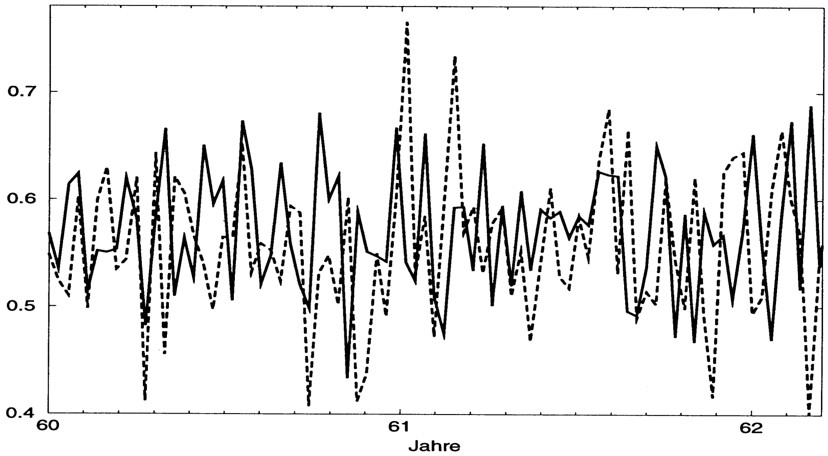


FIGURE 1
Two time series, one of which is generated by a normal random number generator (*solid line*), and the other as a sum of 20 chaotic deterministic processes (*dashed*). The first two moments of the random process are fitted to match the sum of the 20 nonlinear processes (H. von Storch et al., 1999)

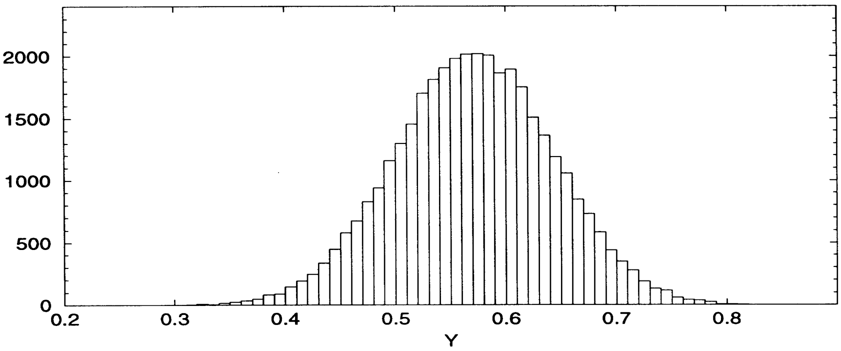


FIGURE 2
Histogram of the time series generated as sum of chaotic processes, shown in Figure 1 (H. von Storch et al., 1999)

not imply that the original system is random in any sense but only that it can be described as random for certain purposes.

The transition from a deterministic description to a stochastic description is motivated by the observation that the behavior of a complex deterministic system is often not distinguishable from the behavior of a stochastic process. This fact is demonstrated in Figure 1. It shows two time series. One represents the sum of 20 chaotic but purely deterministic processes. The other series is a realization of a normal random process, which first two moments match the deterministic time series. While the two series are different at any time instant, their overall character is very similar. The histogram (Figure 1) of the deterministic time series is indeed close to a normal distribution, as a consequence of the Central Limit Theorem. The similarity is usually rationalized by asserting that the evolution of the deterministic system is caused by independent impacts and equating independent impacts with random impacts. Randomness models independence. The exact meaning of these statements is pursued in the foundations of complex system theories. In climate studies, the application of probabilistic concepts has turned out of permitting a suitable description of observed and numerically modeled phenomena.

In climate science, as in many other environmental sciences, two species of models are used: maximum and minimum complexity models (H. von Storch, 2000). They serve different purposes. *Maximum complexity models* attempt to describe the climate system by treating explicitly as many processes as possible – with limits given by the computational facilities. Such models are also labeled simulation models or, in climate sciences, general circulation models (GCMs; see Section 2). They are (almost always) deterministic by construction, but exhibit markedly stochastic behaviour, reflecting the presence of many nonlinear processes. They allow for numerical experimentation and detailed simulation but they do not represent scientific understanding of climate dynamics and climate sensitivity. Instead they produce numbers which need interpretation with the other species of models, namely *minimum complexity models*. In such *cognitive* or *reduced* models only the most relevant processes are considered, while myriads of processes of minor importance are either disregarded or summarily described by random processes. In their attempt to reduce the complex dynamics to a minimum number of interacting processes, such models represent scientific understanding, even if they are unsuited for explaining all details as needed in management decisions. Reduced models may be of dynamical character, like the stochastic climate model (Section 3) or of descriptive character as in the detection problem (Section 4). Stochasticity is invoked in these models to represent the effects of the large variety of disregarded processes which in themselves are rather unimportant but which contribute as a whole significantly to the overall dynamics.

Various sub disciplines get involved in the probabilistic description of the climate system: stochastic differential equations which ignore the detailed structure of the micro-processes and replace them by stochastic processes; and statistics which makes inferences about underlying processes by assuming that the observed climate record or the output of climate simulation models can be treated as a realization of a stationary (or cyclo-stationary) ergodic stochastic process. Standard methods in these sub disciplines, however, often fail to cope with the complexity and heterogeneity of the climate system. New methods, specifically designed for application to the climate system have been and must further be developed.

We illustrate these new developments by two examples: *stochastic climate models* and the *climate change detection problem*. Stochastic climate models presume that weather fluctuations drive climate variability in much the same way that fluid molecules drive Brownian particles (Hasselmann, 1976). The detection problem addresses the question whether the observed increase in surface temperature, or changes in the statistics of other climate variables, is part of the natural climate variability or alternatively related to human activities (Zwiers, 1999).

Before we present our two cases in Sections 3 and 4 in some detail, we briefly review some basic facts about the climate system.

2. Climate System and Climate Models

The fluid flows in the atmosphere and oceans are described by variables like temperature, velocity and density. Climate is described by time averages or ensemble averages of these quantities. Weather represents the time evolutions

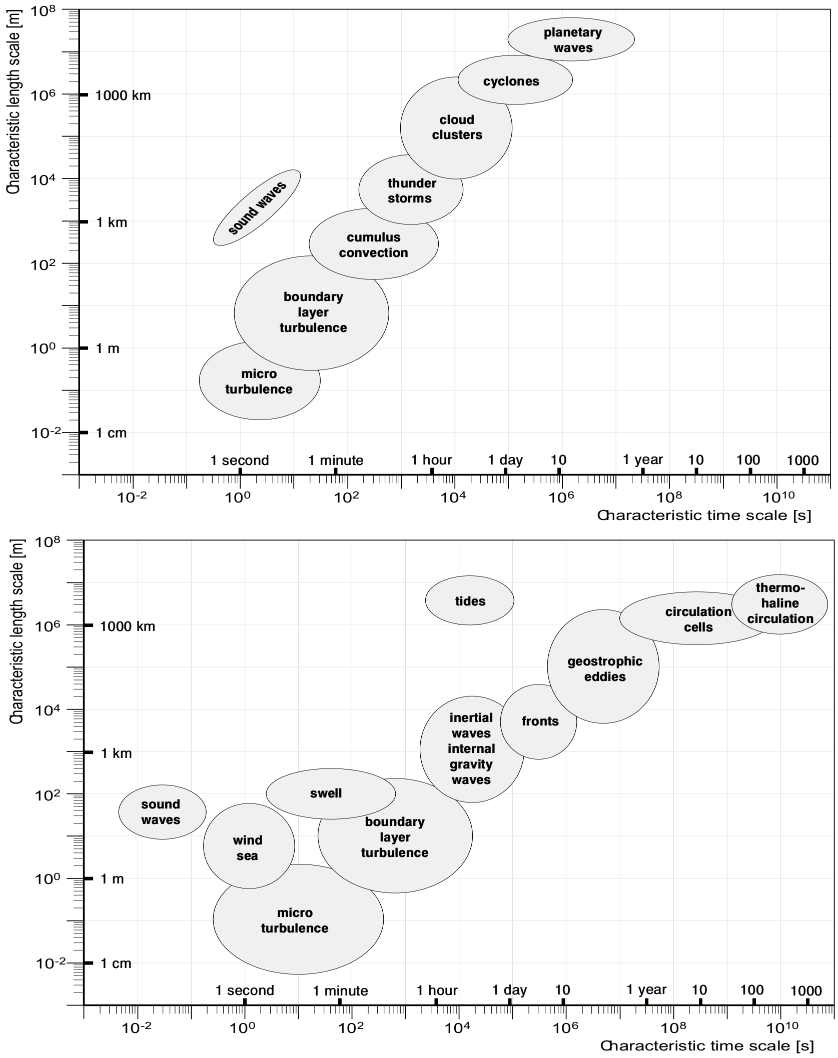


FIGURE 3
Space and time scales of relevant atmospheric and oceanic dynamics processes (H. von Storch and Zwiers, 1999).

of the variables themselves. In this sense, weather is a particular realization of climate.

The climate system does not only consist of the lower part of the atmosphere where we humans live but also includes the upper atmosphere and the oceans. It is a complex and open system. A myriad of processes interact in an open heterogeneous setting, resulting in complex spatial structures and complex temporal evolutions. This is demonstrated by Figure 3 which displays spatial and temporal scales of relevant atmospheric and oceanic processes. In both fluids, dynamical processes generate variability on all time and space scales, while the dynamics depend heavily on the scales.

A large variety of climate models have been developed to study the climate system. Their design and complexity depends on the goal and on the time scales considered. One distinguishes between cognitive (minimum complexity) models and simulation (maximum complexity) models (H. von Storch, 2000). *Cognitive* models aim at understanding of climate dynamics. They ought to be as simple as possible. They describe specific processes of a few degrees of freedom. Energy balance models, featuring incoming and outgoing radiation and their reflection and interception by surfaces, gases and particles, are an example of such cognitive models. The stochastic climate model is another example (Section 3). *Simulation* models aim at scenarios and prognoses. They ought to comprise our current knowledge of climate dynamics as completely as possible. Simulation models do not give immediate insight. Extraction of knowledge requires analysis by statistical methods, cognitive models, and other tools.

Simulation models are numerical models. Current computers allow models with up to 10^7 degrees of freedom. Most simulation models are global coupled Atmosphere Ocean General Circulation Models (AOGCMs). They consist of circulation models for the atmosphere and the ocean. Representations of other components, such as sea-ice, soil and vegetation, are in most cases added. Each model is based on conservation equations for mass, momentum and energy. These equations are solved numerically, using a spatial discretization either in terms of a limited number of spherical harmonics or a limited number of grid points. The models are integrated with a time step of the order of one hour or less for a few days up to hundreds of years at special national and international research centers with access to powerful computing facilities.

A special problem with simulation models is that because of the heterogeneity displayed in Figure 3 no "natural" truncation scale exists. Any truncation disregards processes on scales smaller than the truncation scale: a truncation in an ocean model at 100 km disregards fronts, at 10 km internal waves, and a truncation at 100 m disregards boundary layer turbulence. On the other hand, these disregarded processes have a significant impact on the dynamics of the resolved scales. Therefore the unresolved processes are *parameterized*, i.e., their net effect on the resolved dynamics is taken into account by adding terms describing the impact of the unresolved scales, assuming that this impact on average depends only on the resolved scales. Simulation models feature a large number of such parameterizations, for instance for mixing, convection, and clouds (Washington and Parkinson, 1986). The functional forms of these parameterizations depend on the resolution. They are based on knowledge about processes, empirical evidence and practical considerations. They are very much a subjective choice. Thus, the limiting process, that the mesh sizes converges towards zero, is not defined. The concept of "consistency", fundamental in numerical mathematics, does not apply in this context.

3. Concept of Stochastic Climate Models

The stochastic climate model (Hasselmann, 1976) is a cognitive model, designed not to explain the details of the climate system but to explain the roles of the small number of dominate processes in the formation of climate variability. The concept asserts that weather noise is a constitutive element essential for the formation of internal variability of the slow climate components. Stimulated by the analysis of Brownian motions, Hasselmann suggested that, as molecular fluctuations can supply a Brownian particle with energy, weather noise can supply slow components of the climate system with energy. If this energy supply is turned off, variations of the slow components will eventually die out.

An important result of the analysis of Brownian motions is the fluctuation-dissipation relation, which is expressed in the Langevin equation:

$$\frac{dv}{dt} = -\frac{1}{\tau} v + \zeta \tag{1}$$

where v is the velocity of a one-dimensional Brownian particle, ζ is a white noise that characterizes the molecular fluctuations, and τ the characteristic time scale at which dissipation occurs. The time-scale depends on the mobility and mass of the particle.

When the Brownian particle has gained enough energy to move, it will experience friction. The associated energy loss to the molecules prevents the energy of the particle to grow indefinitely. The fluctuation-dissipation relation states that the energy gained from molecular fluctuations is directly related to the energy lost to molecules. The stronger the fluctuations, the sooner the Brownian particle will gain enough energy to move (corresponding to a smaller τ), and the stronger will be the dissipation experienced by the particle.

The Langevin equation is a stationary stochastic differential equation. The velocity v has an Ornstein-Uhlenbeck spectrum (or a spectrum of a first-order auto regressive process when Eq. (1) is discretized):

$$\Gamma_v(\omega) = \frac{\Gamma_\zeta(\omega)}{1/\tau^2 + (2\pi\omega)^2} \tag{2}$$

where Γ_v and Γ_ζ are the spectra of v and ζ, and ω is the frequency.[2]

The velocity spectrum Γ_v increases with decreasing frequency at the rate of $1/\omega^2$ for $\omega \gg 1/(2\pi\tau)$. This high-frequency spectral increase reflects the fact that the time scale at which energy is gained is much shorter than the time scale at which energy is dissipated. The separation of these two time scales is a key feature of the mechanism. It generates the motion of the Brownian particle.

In their simplest form, stochastic climate models are Langevin equations tailored to the climate system. In this case, v represents a slow degree of freedom of the climate system. ζ describes the fast components that interact with the considered slow components. τ characterizes the time scale at which the slow component has gained enough energy and starts to "move", thereby experiencing friction.

The importance of the above mechanism in generating climate variations is documented by climate spectra which have a $1/\omega^2$ shape at high frequencies. Such spectra are often observed in the climate system (J. S. von Storch et al., 2000b).

A slightly more complex version is introduced when eigenmodes are excited by the noise. In this case a complex version of (1) is considered with a complex

[2] Equation (2) may be derived by calculating the autocovariance function of v and by Fourier transforming the autocovariance function.

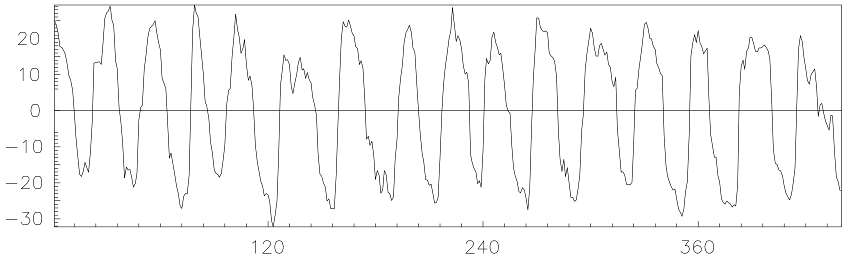


FIGURE 4
Time series of monthly 30-hPa zonal wind for the period 1953–1989 along the equator. The x-axis represents months, beginning with 1 = January 1953 and 120 = December 1962, 240 = December 1972 and 360 = December 1982. Units in *m/sec*.

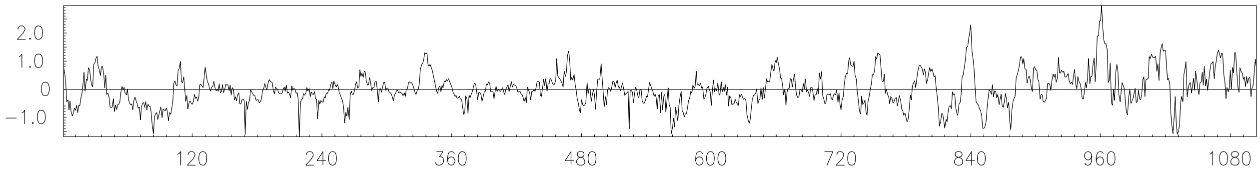


FIGURE 5
Time series of monthly sea surface temperature anomalies averaged over an area in the eastern equatorial Pacific for the period 1903–1994. The x-axis represents months, the units are *K*.

coefficient $\lambda = 1/\tau - i2\pi\omega_o$. ω_o represents the eigen frequency. It can be considered as a general case of (1) in which the eigen frequency is zero. The spectrum of v then takes the form

$$\Gamma_v(\omega) = \frac{\Gamma_\zeta(\omega)}{1/\tau^2 + (2\pi[\omega - \omega_o])^2} \tag{3}$$

If the time scale $1/\omega_o$ is much longer than the time scale of the fluctuations, the spectrum will be proportional to $1/\omega^2$ at high frequencies and will have a peak at ω_o. In general, the weaker the dissipation (i.e. the smaller $1/\tau$), the more pronounced the peak. If there is no dissipation, i.e. $1/\tau = 0$, a resonance peak will be obtained.

Two important large-scale modes[3] of the climate are the quasi-biennial oscillation (QBO) in the lower stratosphere and the El Niño – Southern Oscillation (ENSO) in the tropical Pacific. QBO is related to the downward propagation of the easterlies and westerlies from about 10 hPa to about 100 hPa. This leads to the oscillatory behavior of zonal wind at 30 hPa shown in Figure 4. ENSO is a coupled atmosphere-ocean phenomenon. The warming and cooling in the central and eastern tropical Pacific is associated with the weakening and strengthening of the trade winds and a west-east seesaw behavior of surface pressure in the tropical Indian Ocean and the Pacific. The time series of sea surface temperature shown in Figure 5 characterizes the time evolution of ENSO over the past ninety years. The oscillatory behavior of ENSO is not as pronounced as in the case of the QBO. Figure 6 shows the spectra for the two phenomena. The high-frequency $1/\omega^2$ shape (indicated by the straight line) suggests that both modes can be considered as being excited by short-term fluctuations. In the case of QBO, the fluctuations originate from degrees of freedom representing vertically propagating waves. In the case of ENSO, the fluctuations are mainly related to weather disturbances. The dissipation in the stratosphere seems to be much weaker than that at the air-sea interface, leading to a pronounced peak at the eigen frequency of the QBO, which is about 1 cycle every 28 months, but

[3] A mode represents a motion with well-described spatial structure.

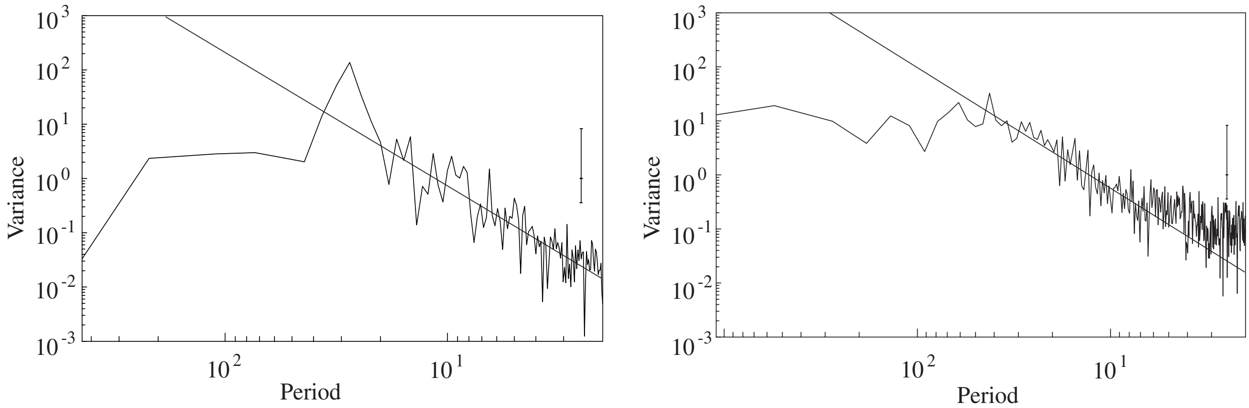


FIGURE 6
Spectra of the QBO and ENSO obtained from the observations shown in Figures 4 (*top*) and 5 (*bottom*). Periods in *months*.

a much weaker peak at the eigen frequency of ENSO, which is about 1 cycle every 3–4 years.

The Ornstein-Uhlenbeck spectrum represents only one type of climate spectra. Being a highly heterogeneous system, the climate system is expected to have variations generated by mechanisms different from those describable by Eq. (1). No matter what mechanisms are operating, fast components represent an important energy source of slow components and must properly be incorporated. An insufficient representation of this source can lead to a different climate variability. This point can be demonstrated by the variability of the deep ocean. The deep ocean is one of the slowest climate components, and can substantially contribute to long-term climate variations. Due to the lack of observational data, the simulation models discussed in Section 2 are the main tool for studying its variations. We will show below that models, which represent the fast components (i.e., the eddies) differently, produce distinctly different oceanic variations.

Due to the smaller density differences in the ocean, the spatial scale of the most energetic eddies is about one order of magnitude smaller in the ocean than in the atmosphere. Both the atmospheric and oceanic eddies influence variations of the ocean. The atmospheric eddies have their largest effect at or near the surface through the fluxes of momentum, fresh water and heat into the ocean. The oceanic eddies play a crucial role for the generation of variations in the ocean interior. Most ocean models do not resolve eddies. In order to produce a realistic mean circulation, strong horizontal diffusion is invoked in these models. When integrating an oceanic model with prescribed mean vertical fluxes of momentum, heat and fresh water at the surface (with constant annual cycles superimposed on them) but without the fluctuating components of these fluxes, then the resulting state shows simple spatial structures with little or no variations. When the effect of atmospheric eddies is included by coupling an oceanic GCM to an eddy-resolving atmospheric GCM with a resolution of a few degrees then the situation changes dramatically. This is demonstrated by an integration of an AOGCM (J.-S. von Storch et al., 1997) over a time period of one thousand years.

In contrast to integrations with ocean-only models driven by mean fluxes, the ocean simulated by the coupled atmosphere-ocean model exhibits significant variations. Figure 7 displays these variations in terms of spectra of the meridional

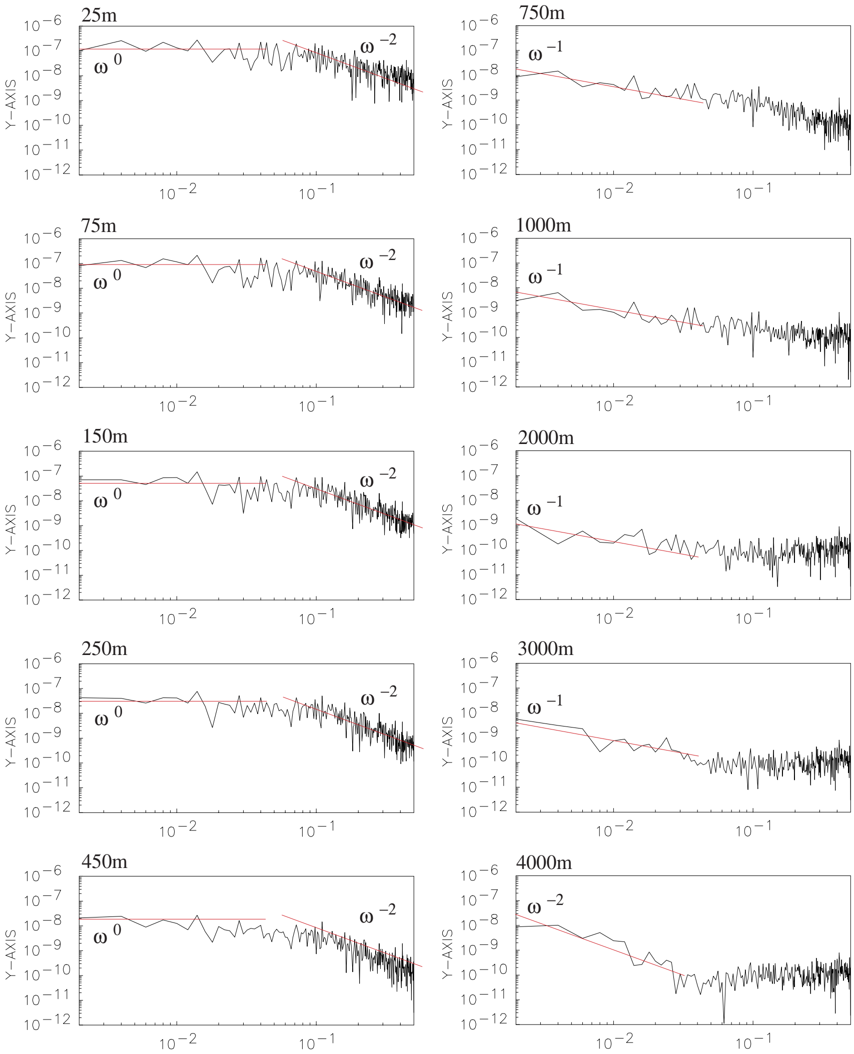


FIGURE 7
Spectra of time series of meridional velocity at (28° W, 18° S) at various depths. The time series are derived from an integration with an AOGCM. The straight lines indicate the spectral slopes. Frequency is in *cycles per year*.

velocity at a grid point in the subtropical South Atlantic at various depths. By coupling the ocean model to an atmospheric model, strong variations are induced.

In the upper few hundred meters of the ocean, the spectra are comparable to an Ornstein-Uhlenbeck spectrum, indicating that the variations are generated in a way similar to that described by the Langevin equation. At greater depth, the tropical and subtropical ocean is stably stratified. The stability efficiently shields the deep ocean from the fluctuating surface fluxes. The variations must be generated in a way different from that of the upper ocean. Indeed, the spectrum at 750 meter depth starts to deviate from an Ornstein-Uhlenbeck spectrum. Below 1000 meter depth, the high-frequency slope reduces to about zero and the low-frequency slope is about minus one to minus two. A new type of spectrum emerges. The variations can no longer be described by the simple Langevin equation. This new type of spectra is also obtained from millennium integrations with other coupled AOGCMs (J.-S. von Storch et al. 2000a).

The variations shown in Figure 7 are obtained with a model not resolving oceanic eddies. It is conceivable that oceanic eddies are so strong that they supply energy to the large-scale oceanic motions and alter their behavior. In this case, variations of the deep ocean circulation may be different from those shown in Figure 7.

4. Filtering Anthropogenic Climate Change from Natural Variability

In the previous section, the stochasticity of the climate system appeared as a constitutive element essential for the formation of internal climate variability unrelated to external forcing factors. However, when climate data are analyzed then the noise appears as concealing the "signals". For instance, Hansen and Sutera (1986) have suggested that the atmospheric circulation in the North Atlantic Sector has a bimodal distribution, with a "zonal" regime bringing storms to Europe, and a "blocked" regime rerouting these storms. However, because of the strong inherent variability of any circulation index, data of about 140 years would be needed to reject the null hypothesis that bi-modality of the distribution would not be a sample coincidence (Nitsche at al., 1994). Thus, in such cases, noise conceals the structure, and statistical techniques are required to filter the signal from the "sea of climatic noise".

The problem of discriminating structures of interest ("signals") from a large number of other, mostly unrelated processes ("noise") is a standard problem in atmospheric and other environmental sciences. Of course, "interest" is changing among different studies and applications so that a process (like ENSO) may sometimes be a signal (for seasonal climate predictions) or noise (for anthropogenic climate change). The separation of signal and noise is usually approached with the classical arsenal of statistical inference, like hypothesis testing, analysis of variance, time filtering, or pattern recognition (for an overview, refer to H. von Storch and Zwiers, 1999).

While statistical inference was in the past a methodical aspect within climate science, it entered the public debate in the early 1990s. The "detection"-question was raised whether the observational record provides evidence about recent climate changes being inconsistent with natural variability. The "attribution"-question was whether such a change could be traced back to specific anthropogenic causes, like the ongoing human emission of greenhouse gases into the atmosphere.

In the following we demonstrate how the detection problem may be addressed (Hegerl et al., 1996). The question is cast as a statistical test with the null hypothesis that the observed recent climate change is entirely within the range of natural climate variability. The alternative hypothesis, then, is that the recent climate change is not entirely within this range, or, in other words, part of the recent change is anthropogenic. The alternative hypothesis is not, that this is due to enhanced greenhouse gas concentrations. This is the attribution problem.

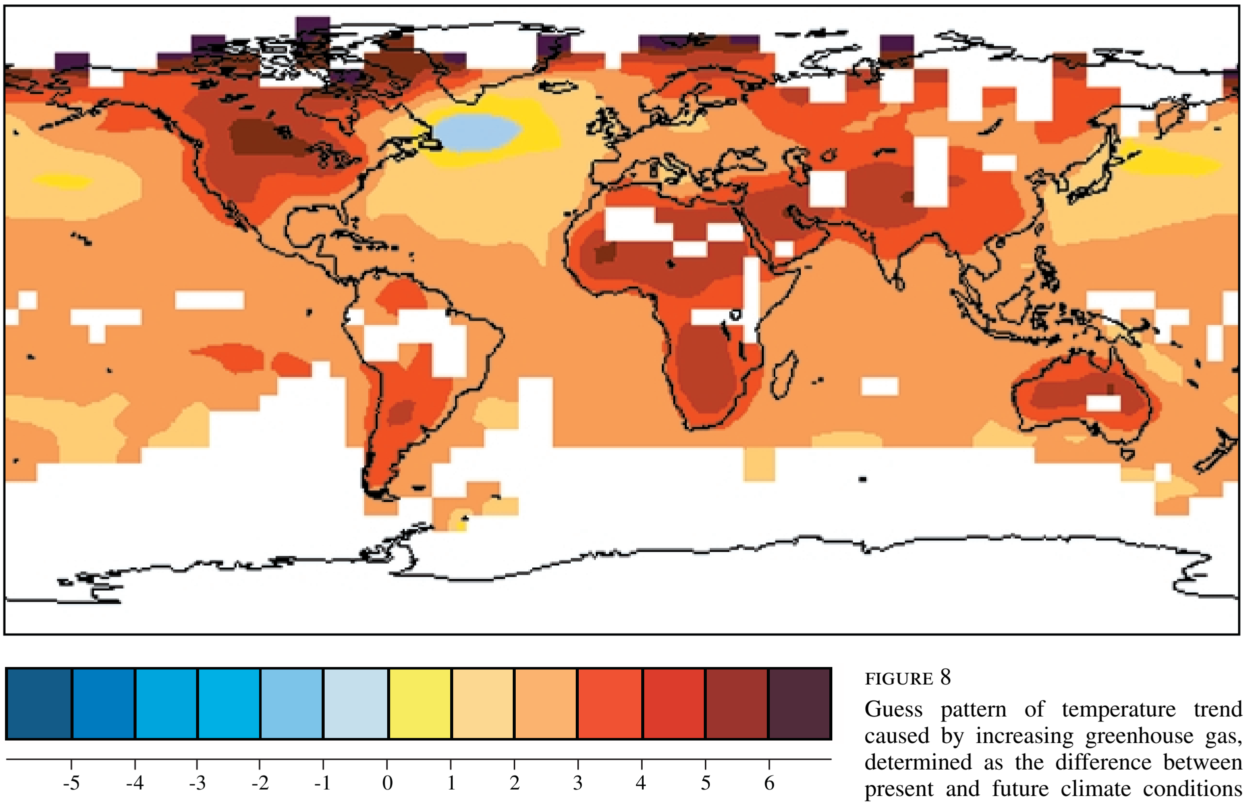


FIGURE 8
Guess pattern of temperature trend caused by increasing greenhouse gas, determined as the difference between present and future climate conditions simulated by a climate model. No units. (Courtesy: Gabriele Hegerl).

The observational record features a wide range of variables, such as air temperatures near the surface and throughout the troposphere, ocean temperature at the surface and in the deep ocean, wind and currents, relative humidity and salinity, cloudiness and ice coverage etc. Inclusion of all these variables would be unreasonable for a number of reasons. Some of the variables have been monitored only for a limited time and with limited spatial coverage; the measurements may have undergone changing quality standards (homogeneity problem), introducing artificial trends into the data record.

The air temperature at the surface is a data set that extends more than 100 years back in time, has adequate (but certainly not ideal) spatial coverage and controlled quality. The decision to deal with surface air temperature reduces the dimensionality of the problem considerably.

Because of the spatial heterogeneity of the temperature field, the dimension of variable is still large – too large for discriminating between noise and the signal of global warming. We demonstrate this problem with a simple example. Consider a n-dimensional random vector with components independent of each other, with Gaussian distribution, zero mean and unit standard deviation. Furthermore, assume that the first component includes a signal so that its expectation is s. Then, the expected squared length of the vector is $T = s^2 + n$. For the null hypothesis of $s = 0$ the squared length would be n, so that the signal-to-noise ratio is s^2/n. The larger the dimension n, the smaller the chances of

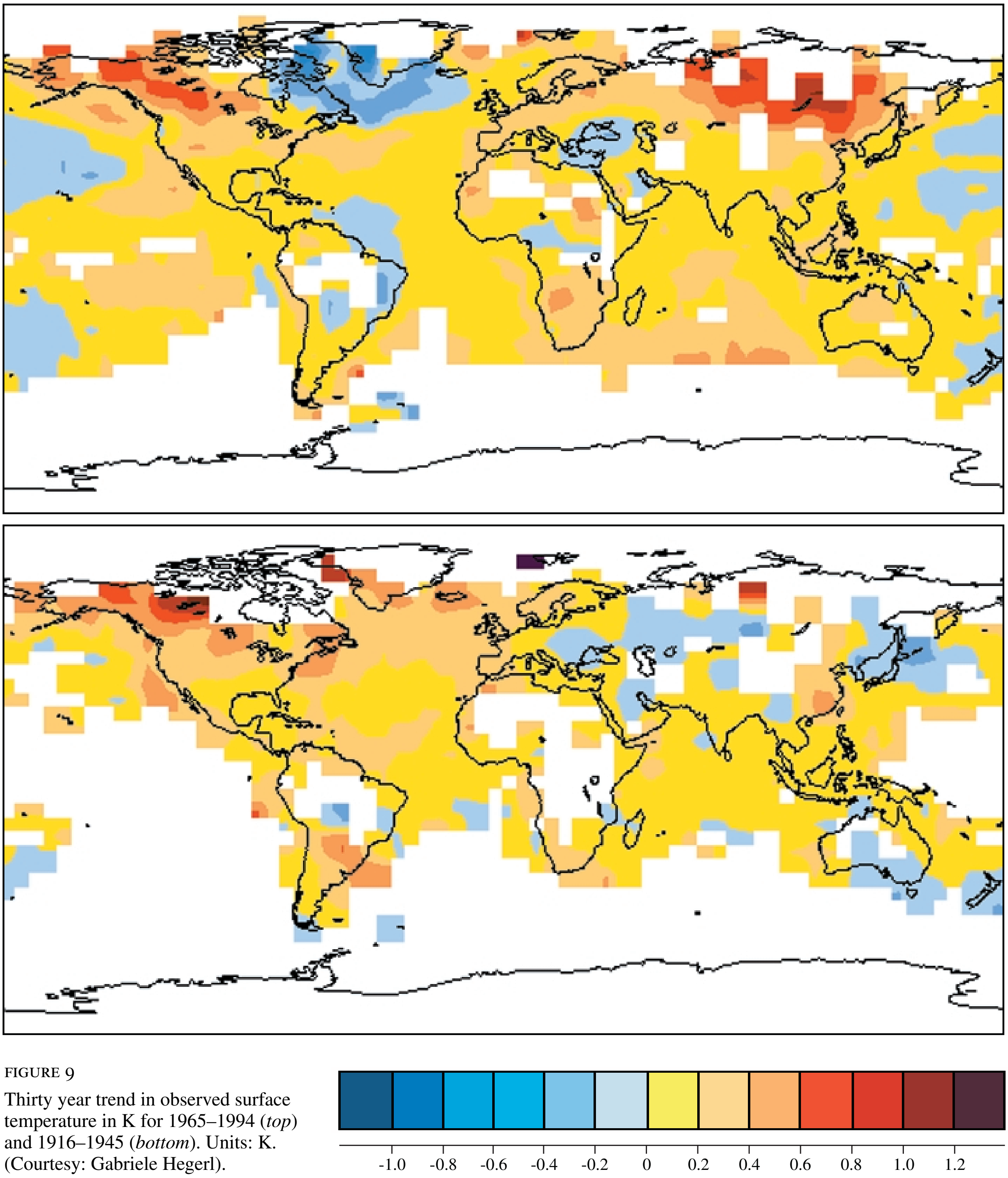


FIGURE 9

Thirty year trend in observed surface temperature in K for 1965–1994 (*top*) and 1916–1945 (*bottom*). Units: K. (Courtesy: Gabriele Hegerl).

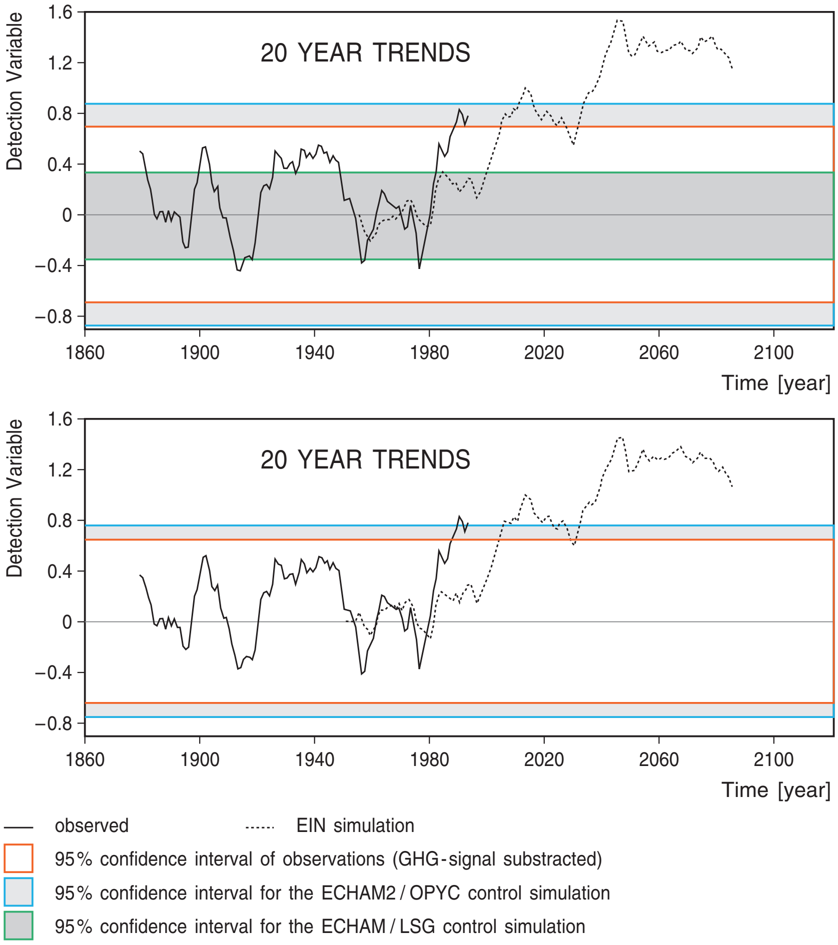


FIGURE 10
Projection of the time series of observed trends on $\boldsymbol{e}^1$ (*a solid line*). The projection of temperature trends simulated in a climate change computation are shown for comparison as dotted line. The time, given at the horizontal axis represents the end-year of 20-year intervals for which the temporal trends are calculated. The horizontal bands are 2σ confidence intervals of natural variability derived from observed data and two climate model simulations. (H. von Storch et al., 1999)

detecting an $s \neq 0$. Thus, a-priori reduction of the dimension is mandatory (Hasselmann, 1979).

This reduction is accomplished by projecting the spatial distribution of the temperature $\boldsymbol{T}$ onto a limited number of "patterns" $\boldsymbol{e}^i$: $\boldsymbol{T} \approx \sum_i \alpha_i \boldsymbol{e}^i$. Then, the analysis is done in the linear space spanned by the first few α_i coefficients. Usually less than 10 of these coefficients are used, in many cases even just one.

The patterns may be chosen in different manners. Candidates are vectors constructed to efficiently represent the variability ("Empirical Orthogonal Functions", known as "principal vectors" in other disciplines), or "guess patterns" considered likely to carry the signal. Such patterns can be derived from climate models. For instance, experiments with climate models (e.g., Cubasch et al., 1995) indicate that the ongoing increase of greenhouse gas concentrations in the atmosphere will lead to a general warming of the air near to the surface, with stronger warming over the continents and delayed warming over the ocean; the delay is, according to the model, particularly evident over the northwestern

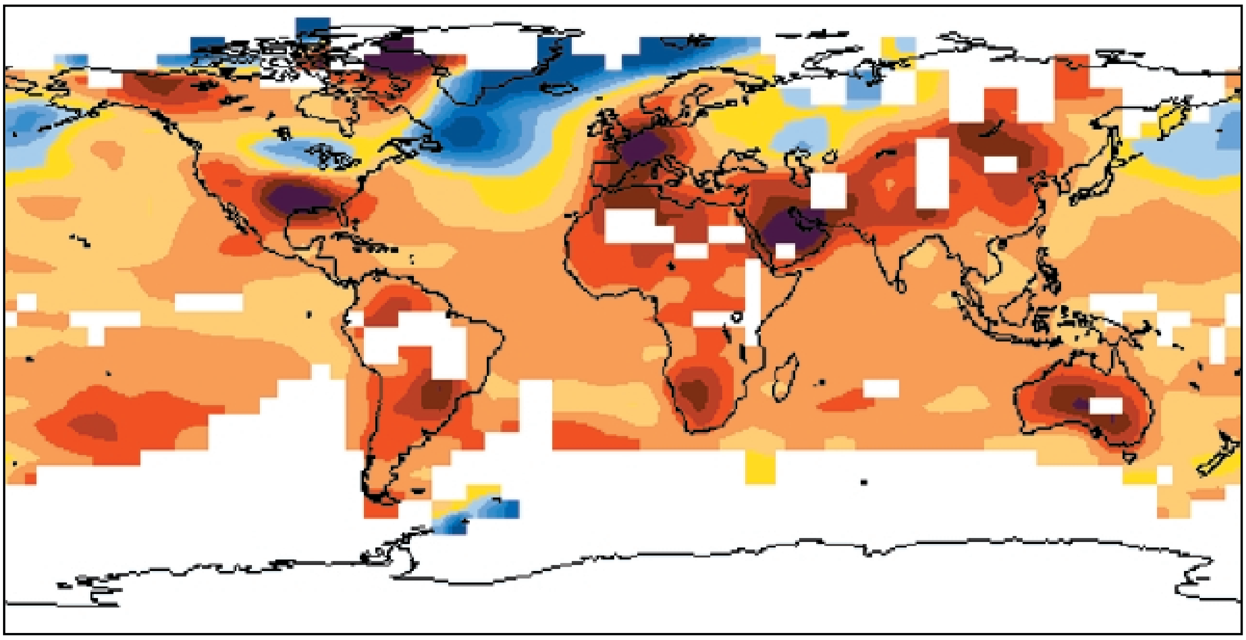
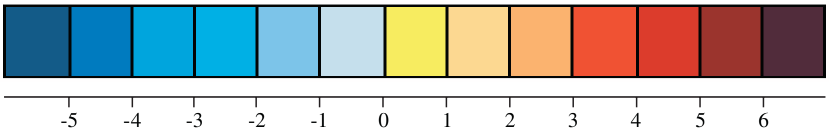


FIGURE 11
Optimized “fingerprint” $q^1 = e^1 \Sigma^{-1}$.
No units. (Courtesy: Gabriele Hegerl).



part of the North Atlantic. Figure 8 displays the warming as calculated by the climate model after 100 years of continuous increase (about 1% per year) of the CO_2 concentration. The moving 20-year temperature trend T_{tr} is projected on to this pattern e^1. Two such trends are displayed in Figure 9. Areas with insufficient data coverage are blanked out. The guess pattern in Figure 8 is also left blank in data sparse areas, for consistency.

The projection of the observed trends on e^1 is displayed in the top diagram of Figure 11 as a solid line. The horizontal bands represent approximately 95% confidence intervals derived from observations and two extended simulations. For the most recent time, the detection variable $T_{tr} \cdot e^1$ leaves the 95% confidence limit as determined from observations and from one climate model. This behavior is to be expected. When the procedure is applied to the simulated climatic response to increasing greenhouse gas concentrations, the dashed curve emerges. It exhibits the same irregular behavior with a general tendency of leaving the confidence interval and large swings, reflecting natural variations. Because of the ubiquitous noise in the climate system, the timing of leaving the confidence interval becomes a random variable itself.

The chances for successfully discriminating between natural variations and systematic changes due to human activities can be enhanced by a formalism to increase the signal to noise ratio. To do so, an educated guess of the signal e^1, like Figure 8, and of the covariance structure Σ are needed. It can be shown

(Hasselmann, 1979, 1993) that the "fingerprint" $\boldsymbol{q}^1 = \boldsymbol{e}^1 \Sigma^{-1}$ optimizes the signal-to-noise ratio, conditional upon the quality of the estimated quantities $\boldsymbol{e}^1$ and Σ. In the present example, the covariance matrix Σ is estimated with data from one of the model simulations, which then no longer provides data for an unbiased estimation of the 95% confidence band. The fingerprint pattern is shown in Figure 10, and the optimized detection variable $\boldsymbol{T}_{\mathrm{tr}} \cdot \boldsymbol{q}^1$ in the lower panel of Figure 11. The optimized fingerprint is not very different from the raw guess pattern; the land-sea contrast is a bit enhanced, but the minimum in the NW Atlantic is unchanged. The optimization has in fact improved the signal-to-noise ratio, even if only slightly.

The procedure just discussed is compromised by the fact that the hypothesis to be tested -that the recent temperature trend is not entirely natural- is in part based on the same data, which is used to test the hypothesis (the "Mexican-Hat" problem, cf. H. von Storch and Zwiers, 1999). To make this subjective element explicit, it is worthwhile to formulate the problem in a Bayesian manner, as suggested by Hasselmann (1998) and Risbey et al. (2000).

5. Conclusion and Outlook

We have discussed the constructive and the concealing role of "noise" in climate dynamics. In a strict sense, the climate system is deterministic, but the practically infinite number of non-linear processes transform the omnipresent small uncertainties into a cacophony indistinguishable from the mathematical construct of random noise. The "noise" is ubiquitous in the climate system; it emerges at all locations and times, at all scales. Our understanding of climate dynamics and our interpretation of the climate record must therefore take into account this peculiarity of the climate system. Climate must be considered a stochastic system, and our climate simulation models as random number generators.

For that reason, climate research is an academic environment which has been abundant in generating stochastic ideas and statistical techniques. The two cases discussed in some details in this paper, the stochastic climate model and the detection problem, are just examples. Other examples of stochastic or statistical methods specific to climate research are: principal oscillation and interaction patterns (POPs and PIPs) which attempt an optimal representation of climate variability (H. von Storch and Zwiers, 1999); data assimilation which attempts to optimally combine data and dynamical models for hind-, now-, and forecasts (Robinson et al., 1998); "potential predictability" which attempts to discriminate low frequency variability arising from the integration of day-to-day noise from genuinely low frequency dynamics (H. von Storch and Zwiers, 1999). So far, most of the statistical analysis done in climate research is "frequentist" in character, while Bayesian views are now understood of being particularly valuable for the specification of parameterizations of sub-grid-scale processes in climate models and for data assimilation.

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